Arithmetic Progressions: Exercise 5.2

Q.1 Fill in the blanks in the following table, given that a is the first term, d the common difference and an the nth term of the AP:

	a	d	n	a_n
(i)	7	3	8	...
(ii)	−18	...	10	0
(iii)	...	−3	18	−5
(iv)	−18.9	2.5	...	3.6
(v)	3.5	0	105	...

Sol. Fill in the blanks:

(i) Since, $a_n = a + (n - 1)\, d$

$\Rightarrow \quad = 7 + (8 - 1)\, 3$

$\Rightarrow \quad = 7 + 7 \times 3 = 7 + 21 = 28$

(ii) Since, $a_n = a + (n - 1)\, d$

$\Rightarrow \quad 0 = -18 + (10-1)\, d$

$\Rightarrow \quad 18 = 9d$

$\Rightarrow \quad d = 18/9 = 2$

(iii) Since, $a_n = a + (n - 1)\, d$

$\Rightarrow -5 = a + (18-1)\, (-3)$

$\Rightarrow -5 = a + 17\, (-3)$

$\Rightarrow -5 = a - 51$

$\Rightarrow a = -5 + 51 = 46$

(iv) Since, $a_n = a + (n - 1)\, d$

$\Rightarrow \quad 3.6 = -18.9 + (n-1)\, 2.5$

$\Rightarrow \quad 3.6 + 18.9 = (n-1)\, 2.5$

$\Rightarrow \quad 22.5 = (n-1)\, 2.5$

$\Rightarrow \quad n-1 = 22.5 / 2.5$

$\Rightarrow \quad n-1 = 9$

$\Rightarrow \quad n = 9+1 = 10$

(v) Since, $a_n = a + (n - 1)\, d$

$= 3.5 + (105-1) \times 0$

$= 3.5 + 0 = 3.5$

	a	d	n	a_n
(i)	7	3	8	28
(ii)	−18	2	10	0
(iii)	46	−3	18	−5
(iv)	−18.9	2.5	10	3.6
(v)	3.5	0	105	3.5

Q.2 Choose the correct choice in the following and justify:

(i) 30th term of the 10, 7, 4,..... is

(a) 97 (b) 77 (c) – 77 (d) – 87

(ii) 11th term of the −3, −12, 2..., is

(a) 28 (b) 22 (c) – 38 (d) $-48\frac{1}{2}$

Sol. **(i) A.P.: 10, 7, 4,.....**

Here, a = 10, d = 7 – 10 = – 3, n = 30

Since, $a_n = a + (n - 1)\,d$

So, $a_{30} = 10 + (30 - 1)(-3)$

$= 10 + 29\,(-3)$

$= 10 - 87 = -77$

Correct Option: (C)

(ii) A.P.: $-3, -\frac{1}{2}, 2...,$

Here, $a = -3$, $d = -\frac{1}{2} - (-3)$

$= -\frac{1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$

n = 11

Since, $a_n = a + (n - 1)\,d$

Therefore, $a_{11} = -3 + (11-1)52$

$= -3+10\times52$

$= -3+25$

$= 22$

So, Correct option: (B).

Q.3 In the following APs, find the missing terms in the boxes :

(i) 2, ☐, 26 **(ii) ☐, 13, ☐, 3**

(iii) 5, ☐, ☐, $9\frac{1}{2}$ **(iv) –4, ☐, ☐, ☐, ☐, 6**

(v) ☐, 38, ☐, ☐, ☐, –22.

Sol. **(i)** Given: A.P.: 2, □, 26,

Since, A.P.: a, (a + d) and (a + 2d)

By comparing, a = 2 and a + 2d = 26

⇒ 2 + 2d = 26

⇒ 2d = 26 – 2 = 24

⇒ d=24/2=12

Hence, the missing term = a + d = 2 + 12 = 14

(ii) Given A.P.: □, 13, □, 3

Since, A.P.: a, a + d, a + 2d and a + 3d

By comparing, a + d = 13 ... (i)

and, a + 3d = 3 ... (ii)

Subtract (i) from (ii)

2d = – 10

⇒ d = – 5

Putting d = – 5 in (i), we get

a – 5 = 13

⇒ a = 13 + 5 = 18

Thus, The missing terms are a, i.e., 18

And, a + 2d, i.e., 18 + 2 (– 5) = 18 – 10 = 8

(iii) Given: A.P.: $5, \square, \square, 9\frac{1}{2},$

Since, A.P.: a, a +d, a +2d and a + 3d.

By comparing, a = 5 ... (i)

and, $a+3d= 9\frac{1}{2} = \frac{19}{2}$ ... (ii)

Subtract (i) from (ii)

$$3d=\frac{19}{2}-5 = \frac{9}{2}$$

$$\Rightarrow d = \frac{1}{3} \times \frac{9}{2} = \frac{3}{2}$$

Thus, The missing terms are a + d, i.e.,

$$5+\frac{3}{2} = 5 + 1\frac{1}{2} = 6\frac{1}{2}$$

and $a + 2d, 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

(iv) Given: A.P.: – 4, □, □, □, □, 6

Since, A.P.: a, a + d, a + 2 d, a + 3 d, a + 4 d and a + 5 d.

By comparing, a = – 4 ... (i)

and, a + 5d = 6 ... (ii)

Subtract (i) from (ii),

5d = 10

d = 2

Thus, the missing terms:

a + d = – 4 + 2 = – 2

a + 2d = – 4 + 2 × 2 = – 4 + 4 = 0,

a + 3d = – 4 + 3 × 2 = – 4 + 6 = 2,

and, a + 4d = – 4+ 4 × 2 = – 4 + 8 = 4

(v) Given: A.P.: □, 38, □, □, □, – 22

Since, A.P.: a, a + d, a + 2 d, a + 3 d, a + 4 d and a + 5 d.

By comparing, a + d = 38 ... (i)

and a + 5d = – 22 ... (ii)

Subtract (i) from (ii),

$4d = -60$

$\Rightarrow d = -15$

Put $d = -15$ in (i), we get

$a - 15 = 38$

$\Rightarrow 38 + 15 = 53$

Thus, The missing terms: $a = 53$.

$a + 2d$, i.e., $53 + 2 \times (-15) = 53 - 30 = 23$,

$a + 3d$, i.e., $53 + 3 \times (-15) = 53 - 45 = 8$,

and $a + 4d$, i.e., $53 + 4 \times (-15) = 53 - 60 = -7$

Q.4 Which terms of the AP: 3, 8, 13, 18, ... is 78 ?

Sol. Given: AP: **3, 8, 13, 18, ...**

From above, $a = 3$, $d = 8 - 3 = 5$

Let 78 be the nth term of the given AP:

$a_n = 78$

Since, $a_n = a + (n - 1) d$

$\Rightarrow a + (n-1) d = 78$

$3 + (n-1) 5 = 78$

$\Rightarrow (n-1) 5 = 78 - 3$

$\Rightarrow 5(n-1) = 75$

$\Rightarrow n-1=15$

$\Rightarrow n=15+1$

$\Rightarrow n=16$

Therefore, 78 is the 16^{th} term of the given A.P.

Q.5 Find the number of terms in each of the following APs:

(i) 7, 13, 19 ...., 205

(ii) 18, 1512, 13, ..., – 47

Sol. **(i)** Given A.P.: 7, 13, 19 ...., 205

From above, First term, $a = 3$

Common difference, $d = 13 - 7 = 6$

Let n be terms in the given A.P. Then,

n^{th} term = 205

Since, $a_n = a + (n - 1) d$

$\Rightarrow a + (n-1) d = 205$

$\Rightarrow 7 + (n-1) 6 = 205$

$\Rightarrow 6 (n - 1) = 205 - 7$

$\Rightarrow 6 (n - 1) = 198$

$\Rightarrow n-1 = 198/6 = 33$

$\Rightarrow n = 33 + 1 = 34$

Therefore, the given A.P. has 34 terms.

(ii) Given A.P.: $18, 15\frac{1}{2}, 13, ..., -47$.

First term, $a = 18$

Common difference, $d = 15\frac{1}{2} - 18$

$= \frac{31}{2} - 18 = -\frac{5}{2}$

Since, n^{th} term = – 47.

$\Rightarrow a + (n - 1) d = -47$

$\Rightarrow 18 + (n-1)(-\frac{5}{2}) = -47$

$\Rightarrow (-\frac{5}{2})(n-1) = -47-18$

$\Rightarrow n-1 = -65 \times -\frac{2}{5}$

$n-1 = -13 \times -2$

$\Rightarrow n = 26 + 1 = 27$

So, the given A.P. has 27 terms.

Q.6 Check whether – 150 is a term of the AP: 11, 8, 5, 2, ...

Sol. **Given:** A.P.: 11, 8, 5, 2,....

Here, $a_2 - a_1 = 8-11= -3$

$a_3 - a_2 = 5 - 8 = -3$

$a_4 - a_3 = 2 - 5 = -3$

Since, common difference, $d = a_{n+1}-a_n$ is same every time, so the given sequence forms an AP.

Now, $a = 11, d = -3$

Let – 150 be the n^{th} term of the A.P.

Since, $a_n = a + (n - 1) d$

$\Rightarrow -150 = 11 + (n-1)(-3)$

$\Rightarrow -3(n-1) = -150 -11= -161$

$\Rightarrow n-1 = 161/3$

$\Rightarrow n = 161/3 + 1=164/3$

The value of n should be a positive integer. So, - 150 is not a term of the given AP.

Q.7 Find the 31st term of an AP whose 11 th term is 38 and the 16th term is 73.

Sol. Let a be the first term and d be the common difference of the A.P.

Since, $a_n = a + (n - 1) d$

Given: $a_{11} = a + 10d = 38$ ... (i)

and $a_{16} = a + 15d = 73$ ... (ii)

Subtract (i) from (ii),

$5d = 35$

$\Rightarrow d = 35/5 = 7$

Put in (i)

$a + 10 \times 7 = 38$

$\Rightarrow a = 38 - 70 = -32$

So, 31^{th} term, $a_{31} = a + 30d = -32 + 30 \times 7$

$= -32 + 210 = 178$

Q.8 An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

Sol. Let a be the first term and d the common difference of the A.P.

Since, $a_n = a + (n - 1) d$

$a_3 = a + 2d = 12$ ... (i)

$a50=a+49d=106$ ... (ii)

Subtracting (i) from (ii)

$47d = 94$

$\Rightarrow d = 94/47 = 2$

and then from (i)

$a + 2 \times 2 = 12$

$\Rightarrow a = 12 - 4 = 8$

So, 29^{th} term, $a_{29} = a + 28 d$

$= 8 + 28 \times 2$

$= 8 + 56 = 64$

Q.9 If the 3rd and 9th terms of an AP are 4 and 8 respectively, which term of this AP is zero?

Sol. Let a be the first term and d the common difference of the A.P.

Since, $a_n = a + (n - 1)\,d$

$a_3 = a + 2d = 4$ ... (i)

$a_9 = a + 8d = -8$ ... (ii)

Subtracting (i) from (ii)

$6d = -12 \Rightarrow d = -12/6 = -2$

Put $d = 2$ in (i)

$a + 2 \times (-2) = 4$

$\Rightarrow a = 4 + 4 = 8$

Let $a_n = 0$

$\Rightarrow a + (n - 1)\,d = 0$

$\Rightarrow 8 + (n - 1)(-2) = 0$

$\Rightarrow (n - 1)(-2) = -8$

$\Rightarrow n-1 = -8/-2 = 4$

$\Rightarrow n = 4 + 1 = 5$

So, 5th term of the A.P. is zero.

Q.10 The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Sol. Let a be the first term and d the common difference of the A.P.

given: $a_{17} - a_{10} = 7$

Since, $a_n = a + (n - 1)\,d$

$\Rightarrow (a + 16d) - (a + 9d) = 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

Thus, the common difference of the A.P. is 1.

Q.11 Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term?

Sol. Given: AP: 3, 15, 27, 39, ...

From above, $a = 3$, $d = 15 - 3 = 12$.

Since, $a_n = a + (n - 1)\,d$

According to question:

$a_{54} = a + 53d$

$= 3 + 53 \times 12 = 3 + 636 = 639$

Let an be 132 more than its 54th term

$a_n = a_{54} + 132 = 639 + 132 = 771$

$\Rightarrow a + (n - 1)\,d = 771$

$\Rightarrow 3 + (n - 1)12 = 771$

$\Rightarrow 12(n - 1) = 771 - 3$

$\Rightarrow 12(n - 1) = 768$

$\Rightarrow n-1 = 768/12 = 64$

$\Rightarrow n = 64 + 1 = 65$

Therefore, 65th term is 132 more than its 54th term.

Q.12 Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

Sol. Let the two APs

A.P.$_1$: $a_1, a_2, a_3, \ldots a_n$, and A.P.$_2$: $b_1, b_2, b_3 \ldots, b_n$

Also, let d be the common difference of two these APs.

$a_n = a_1 + (n - 1)\,d$

and $b_n = b_1 + (n-1)\,d$

$\Rightarrow a_n - b_n = [a_1 + (n-1)\,d] - [b_1 + (n-1)\,d]$

$\Rightarrow a_n - b_n = a_1 - b_1$ for all $n \notin N$

The difference between their 100th term is 100:

$\Rightarrow a_{100} - b_{100} = a_1 - b_1 = 100$

Now, $a_{1000} - b_{1000} = a_1 - b_1$

$\Rightarrow a_{1000} - b_{1000} = 100$ [Since $a_1 - b_1 = 100$]

Thus, the difference between 1000th terms is the same as the difference between 100th terms of given Aps.

Q.13 How many three-digit numbers are divisible by 7?

Sol. **Since,** 105 is the first 3-digit and 994 is the last 3-digit number, which are divisible by 7. Therefore, we have to determine the number of terms in the sequence of 105, 112, 119, ... 994. Which form A.P.

First term, a = 105 and common difference, d = 112 – 105 = 7

Let, n^{th} term = 994.

Since, $a_n = a + (n-1)\,d$

$\Rightarrow 105 + (n-1)7 = 994$

$\Rightarrow 7(n-1) = 994 - 105$

$\Rightarrow 7\,(n-1) = 889$

$\Rightarrow n-1 = 889/7 = 127$

$\Rightarrow n = 127 + 1 = 128$

Thus, there are 128 numbers of three digit which are divisible by 7.

Q.14 How many multiples of 4 lie between 10 are 250 ?

Sol. As we know that 12 is the first integer, which is divisible by 4 between 10 and 250. Also, 248 is the largest integer divisible by 4 between 10 and 250.

12, 16, 20 248

This sequence form A.P. with first term = 12, last term = 248, and common difference = 4 (as the numbers are divisible by 4).

Let n be the number of terms in the A.P.

$a_n = 248$

$\Rightarrow 12 + (n-1)\,4 = 248$

$\Rightarrow 4(n-1) = 248 - 12$

$\Rightarrow 4(n-1) = 236$

$\Rightarrow n-1 = 236/4 = 59$

$\Rightarrow n = 59 + 1 = 60$

Therefore, there are 60 multiples of 4 lie between 10 and 250.

Q.15 For what value of n, are the n^{th} terms of the APs : 63, 65, 67, ... and 3, 10, 17,... are equal?

Sol. Let n^{th} terms of the APs 63, 65, 67, ... and 3, 10, 17, ... are equal.

Since, in 1st AP, $a_1 = 63$, $d_1 = 65 - 63 = 2$ and in 2nd AP, $a_2 = 3$, $d_2 = 10 - 3 = 7$

Since, $a_n = a + (n-1)\,d$

Then,

$63 + (n-1)\,2 = 3 + (n-1)\,7$

$\Rightarrow 7(n-1) - 2\,(n-1) = 63 - 3$

$\Rightarrow (n-1)\,(7-2) = 60$

$\Rightarrow 5(n-1) = 60$

$\Rightarrow n-1 = 60/5 = 12$

$\Rightarrow n = 12 + 1 = 13$

Thus, the 13^{th} terms of the two given APs are equal.

Q.16 Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol. Let a be the first term and d the common difference of the A.p.

Since, $a_3 = 16$ and $a_7 = 12 + a_5$

$\Rightarrow a + 2d = 16$ ... (i)

and, $(a + 6d) - (a + 4d) = 12$

$\Rightarrow 2d = 12$

$\Rightarrow d = 6$ ... (ii) put this in (i)

$a + 2 \times 6 = 16$

$\Rightarrow a = 16 - 12 = 4$

Therefore, the A.P. is $4,\ 4 + 6,\ 4 + 2 \times 6,\ 4 + 3 \times 6,$ ... i.e.,

A.P.: 4, 10, 16, 22, ...

Q.17 Find the 20th term from the last term of the AP: 3, 8, 13, ... 253.

Sol. We have, l = Last term = 253

and d = Common difference = 8 – 3 = 5

Therefore, 20th term from the end = l = (20 – 1)d

= l – 19 d

= 253 – 19 × 5

= 253 – 95 = 158

Q.18 The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol. Let a be the first term and d the common difference of the A.P.

Since, the sum of the 4th and 8th terms of an AP is 24

$a_4 + a_8 = 24$

$\Rightarrow (a + 3d) + (a + 7d) = 24$

$\Rightarrow 2a + 10d = 24$.....(i)

And the sum of the 6th and 10th terms is 44:

$a_6 + a_{10} = 44$

$\Rightarrow (a + 5d) + (a + 9d) = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22$ ..... (ii)

Subtracting (i) from (ii), we get

$2d = 10$

$\Rightarrow d = 5$, Put in (i)

$a + 25 = 12 \Rightarrow a = -13$

The first three terms are a, (a + d) and (a + 2d)

Thus, A.P.: – 13, (– 13 + 5), (– 13 + 2 × 5)

A.P.: – 13, – 8 and – 3

Q.19 Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Sol. Since, the annual salary drawn by Subha in the years 1995, 1996, 1997, etc. = Rs 5,000, Rs 5,200, Rs 5,400.... Rs 7,000.

The sequence of annual salary: 5000, 5200, 5400, ... 7000 form an AP with first term, a = 5000 and common term, d = 200

Let $a_n = 7000$

$\Rightarrow 7000 = a + (n - 1) d$

$\Rightarrow 7000 = 5000 + (n - 1) (200)$

$\Rightarrow 200 (n - 1) = 7000 - 5000$

$\Rightarrow n-1= 2000/200 =10$

$\Rightarrow n = 10 + 1 = 11$

Therefore, in the 11th year i.e., in year 2005 of his service, She drew an annual salary of Rs. 7,000.

Q.20 Ramkali saves Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Sol. Ramkali's saves the amount of Rs. 5 in the subsequent weeks are Rs 5, Rs 5 + Rs 1.75, Rs 5 + 2 × Rs 1.75, Rs 5 + 3 × Rs 1.75.....

And in the nth week her saving will be:

Rs 5 + (n – 1) × Rs 1.75 20.75=

$\Rightarrow 5 + (n - 1) \times 1.75 = 20.75$

$\Rightarrow (n - 1) \times 1.75 = 20.75 - 5 = 15.75$

Therefore n–1=15.751.75=9

$\Rightarrow n = 9 + 1 = 10$

So, In the 10th week, Ramkali weekly savings become Rs 20.75