Arithmetic Progressions: Exercise 5.1

Q.1 In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in the cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Sol. **(i)** The taxi fare for journey of first 1km 15 Rs.
And Rs 8 for each additional km, so taxi fare for 2 km = 15+8 = Rs. 23
Taxi fare for 3 = 23+8 = Rs. 31
Taxi fare for 4 km =31+8 = R.s39
so, taxi fare sequence ⇒ 15, 23, 31, 39....
Since, here every term continually increases with same difference number 8. So, the sequence forms an AP.

(ii) Let x unit be the amount of air present in the cylinder.
The list giving the air present in the cylinder is given by

$$x,\ x-\frac{x}{4}=\frac{3x}{4},\ \frac{3x}{4}-\frac{1}{4}\times\frac{3x}{4}=\frac{12x-3x}{16}=\frac{9x}{16}\ \ldots\ldots\ldots$$

Differences, $\frac{3x}{4}-x=-\frac{x}{4}$

and $\frac{9x}{16}-\frac{3x}{4}=\frac{9x-12}{16}=\frac{-3x}{16}$

⇒ since here differences are not same. So, these numbers do not form an AP.

(iii) Since, the cost of digging a well for the first metre is 150 Rs. and its succeeding metres and rises by Rs. 50 for each subsequent metre.
So, the cost of digging will be: 150, 200, 250, 300...
Since, here every term continually increase with same number 50. So, the list forms an AP.

(iv) The money in the account in the first year is 10000 Rs. and in succeeding years the amount is given by compound interest at 8% per annum:

$$\Rightarrow 10000,\ 10000\times\left[\frac{1+8}{100}\right],\ 10000\times\left[\frac{1+8}{100}\right]^2\ \ldots..$$

From above we can see that common differences are not same. So, these numbers do not form an AP.

Q.2 Write first four terms of the AP, when the first term a and common difference d are given as follows:

(i) a = 10, d = 10 **(ii) a = – 2, d = 0**
(iii) a = 4, d = – 3 **(iv) a = –1 d = 1/2**
(v) a = – 1.25, d = – 0.25

Sol. Since, if the first term is a and the common difference is d,
Then A.P. Wil be: a, a + d, a + 2d, a + 3d,....

(i) Now, Putting a = 10, d = 10 in sequence a, a + d, a + 2d, a + 3d...
Then, the required AP: 10, 10 + 10, 10 + 20, 10 + 30, ...
A.P: 10, 20 , 30 , 40 ....

(ii) Put a = – 2, d = 0 in sequence a, a + d , a + 2d, a + 3d, ...,
Then, A.P.: – 2, – 2 + 0 – 2 + 2 × 0, – 2 + 3 × 0, ...
A.P., – 2, – 2, – 2, – 2, ...

(iii) Put a = 4, d = – 3 in sequence a, a + d, a + 2d, a + 3d, ...
Then, A.P.: 4, 4 – 3, 4 – 6, 4 – 9, ...
A.P.: 4, 1 , – 2, – 5, ...

(iv) Put a = – 1, d=12 in sequence a + 0, a + 2d, a + 3d, ...
Then, A.P.: −1, −1+12, −1+22, −1+32,.....
A.P.: −1,−12,0,12....

(v) Put a = – 1.25, d = – 0.25 in sequence a, a + d, a + 2d, a + 3d,....
Then, AP: – 1.25, – 1.25 – 0.25, – 1.25 – 0.50, – 1.25 – 0.75, ...
A.P. – 1.25, – 1.50, – 1.75, – 2.00, ...

Q.3 For the following APs, write the first term and the common difference:
(i) 3, 1,−1, −3, ...
(ii) −5, −1, 3, 7, ...
(iii) 13,53,93,133....
(iv) 0.6, 1.7, 2.8, 3.9...

Sol. **(i)** Given AP: 3, 1, – 1, – 3.....
First Term, a = 3 and common difference, d = 1 – 3 = – 2.

(ii) Given AP: – 5, −1, 3, 7, ...
First Term, a = – 5 and Common difference, d = −1 – (−5) = – 1 + 5 = 4

(iii) Given AP: $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$........

First Term, a = $\frac{1}{3}$ and Common difference, d = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) The given AP is 0.6, 1.7, 2.8, 3.9 ...
First Term, a = 0.6 and Common difference, d = 1.7 – 0.6 = 1.1

Q.4 Which of the following are APs? If they form an AP, find the common difference d. and write three more terms.
(i) 2, 4, 8, 16, ...
(ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$,....
(iii) – 1.2, – 3.2, – 5.2, – 7.2, ...
(iv) – 10, – 6, – 2, 2, ...
(v) 3, 3 + $\sqrt{2}$, 3 + 2 $\sqrt{2}$, 2 , 3+3 $\sqrt{2}$,....
(vi) 0.2, 0.22, 0.222, 0.2222, ...
(vii) 0, – 4, – 8, – 12, ...
(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$,....
(ix) 1, 3, 9, 27, ...
(x) a, 2a, 3a, 4a, ...
(xi) a, a^2, a^3, a^4....
(xii) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, ...
(xiii) $\sqrt{3}$, $\sqrt{6}$, $\sqrt{9}$, $\sqrt{12}$,...

(xiv) $1^2, 3^2, 5^2, 7^2$....
(xv) $1^2, 5^2, 7^2, 7^3$....

Sol. **(i)** Given Sequence: 2, 4, 8, 16...
Here, $a_2 - a_1 = 4 - 2 = 2$
and, $a_3 - a_2 = 8 - 4 = 4$
$\Rightarrow$Since, $a_2 - a_1 \neq a_3 - a_2$
Therefore, sequence 2, 4, 8, 16 ... does not form an A.P.

(ii) Given Sequence: **2, $\frac{5}{2}$, 3, $\frac{7}{2}$,....**

Here, $a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$,

and, $a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$

since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = \frac{1}{2}$

the next three terms: $\frac{7}{2} + \frac{1}{2} = 4$,

$4 + \frac{1}{2} = \frac{9}{2}$

and $\frac{9}{2} + \frac{1}{2} = 10/2 = 5$

(iii) Given Sequence: – 1.2, – 3.2, – 5.2, – 7.2...
$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$,
$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$
and $a_4 - a_3 = -7.2 - (-5.2 = -7.2 + 5.2 = -2$
Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.
So, $d = -2$
The next three terms:
$-7.2 - 2 = -9.2$,
$-9.2 - 2 = -11.2$,
and, $-11.2 - 2 = -13.2$

(iv) Given Sequence: **– 10, – 6, – 2, 2, ...**
$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$,
$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$,
and $a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$
Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.
So, $d = 4$
The next three terms:
$2 + 4 = 6$,
$6 + 4 = 10$,
and, $10 + 4 = 14$

(v) Given Sequence: $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 2, 3 + 3\sqrt{2}$,...
$a_2 - a_1 = (3 + \sqrt{2}) - 3 = \sqrt{2}$
and $a_3 - a_2 = (3 + 2\sqrt{2}) - (3 + \sqrt{2}) = \sqrt{2}$
Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = \sqrt{2}$

The next three terms:

$(3+3\sqrt{2}) + \sqrt{2} = 3 + 4\sqrt{2}$

$(3 + 4\sqrt{2}) + \sqrt{2} = 3 + 5\sqrt{2}$

and $(3+5\sqrt{2}) + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) Given Sequence: 0.2, 0.22, 0.222, 0.2222, ...

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

and, $a_3 - a_2 = 0.222 - 0.22 = 0.002$

Therefore $a_2 - a_1 \neq a_3 - a_2$

Here, difference between the two terms is not equal. So, the given sequence does not form an AP.

(vii) Given Sequence: 0, – 4, – 8, – 12, ...

$a_2 - a_1 = -4 - 0 = -4,$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4,$

and $a_4 - a_3 = 12 - (-8) = -12 + 8 = -4,$

Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = -4$

The next three terms:

$-12 + (-4) = -12 - 4 = -16$

$-16 + (-4) = -16 - 4 = -20$

and $-20 + (-4) = -20 - 4 = -24$.

(viii) Given Sequence: $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$

$a_2 - a_1 = -\frac{1}{2} - (-\frac{1}{2})$

$= -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - (-\frac{1}{2})$

$= -\frac{1}{2} + \frac{1}{2} = 0$

and $a_4 - a_3 = -\frac{1}{2} - (-\frac{1}{2})$

$= -\frac{1}{2} + \frac{1}{2} = 0$

Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = 0$

The next three terms:

$-\frac{1}{2}, -\frac{1}{2}$ and $-\frac{1}{2}$

(ix) Given Sequence: 1, 3, 9, 27,....

$a_2 - a_1 = 3 - 1 = 2$

and $a_3 - a_2 = 9 - 3 = 6$

$\Rightarrow a_2 - a_1 \neq a_3 - a_2$

Since, difference between the two terms is not equal. So, the given sequence does not form an AP.

(x) Given Sequence: a, 2a, 3a, 4a,...

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = a$

The next three terms:

$4a + a = 5a$,

$5a + a = 6a$

and, $6a + a = 7a$

(xi) Given Sequence: $a, a^2, a^3, a^4 ....$

$a_2 - a_1 = a^2 - a = a(a-1)$

and $a^3 - a^2 = a^3 - a^2 = a^2(a-1)$

$\Rightarrow a_2 - a_1 \neq a_3 - a_2$

Since, difference between the two terms is not equal. So, the given sequence does not form an AP.

(xii) Given Sequence: $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = \sqrt{4x2} - \sqrt{2}$

$= 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

and, $a_3 - a_2 = \sqrt{18} - \sqrt{8} = \sqrt{9x2} - \sqrt{4x2}$

$= 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = \sqrt{2}$

The next three terms:

$\sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$

$= \sqrt{25x2} = \sqrt{50}$

(xiii) Given Sequence: $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{2x3} - \sqrt{3}$

$= \sqrt{3}\,(\sqrt{2} - 1)$

and, $a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3x3} - \sqrt{2x3}$

$= \sqrt{3}\,(\sqrt{3} - \sqrt{2})$

$\Rightarrow \quad a_2 - a_1 \neq a_3 - a_2$

Since, difference between the two terms is not equal. So, the given sequence does not form an AP.

(xiv) Given Sequence: $1^2, 3^2, 5^2, 7^2 ....$

$a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$

and $a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow a_2 - a_1 \neq a_3 - a_2$

Since, difference between the two terms is not equal. So, the given sequence does not form an AP.

(xv) Given Sequence: $1^2, 5^2, 7^2, 7^3 ....$

$a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$,

$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$,

and $a_4 - a_3 = 73 - 72 = 73 - 49 = 24$.

Since, common difference, $d = a_{n+1} - a_n$ is same every time, so the given sequence forms an AP.

So, $d = 24$

The next three terms:

$73 + 24 = 97$,

$97 + 24 = 121$,

and $121 + 24 = 145$.