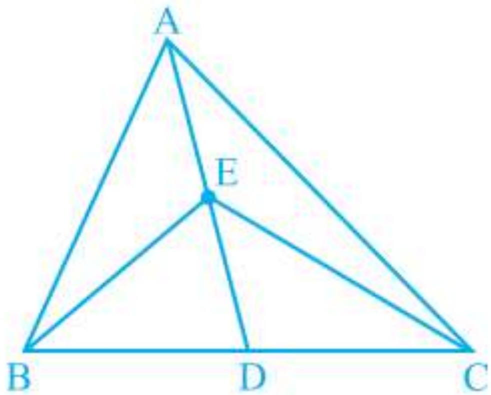
Areas of Parallelogram and Triangles: Exercise 9.3

Q.1 In figure, E is any point on median AD of a Δ ABC. Show that ar (ABE) = ar (ACE).



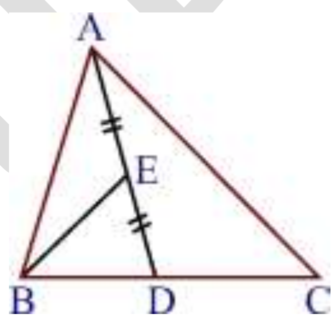
***Sol.* Given:** In ΔABC, AD is a medium and E is any point on AD.
To prove: ar (ABE) = ar (ACE)
Proof: Since given that AD is the median of ΔABC. As we know that median divides a triangle into two triangles of equal area.
So, ar (ABD) = ar (ACD) ... (i)
And also, ED is the median of ΔEBC and median divides a triangle into two triangles of equal area.
So, ar (BED) = ar (CED) ... (ii)
Now, by subtracting (ii) from (i),
ar (ABD) – ar (BED) = ar (ACD) – ar (CED)
$\Rightarrow$ ar (ABE) = ar (ACE)
Hence Proved.

Q.2 In a triangle ABC, E is the mid- point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).

***Sol.* Given:** In a ΔABC, E is the mid- point of the median AD.

To prove: ar (BED) = $\frac{1}{4}$ ar (ABC)

Construct: Join EB.



Proof: Since given that AD is a median of ΔABC and as we know that median divides a triangle into two triangles of equal area.
So, ar (ABD) = ar (ADC)

$\Rightarrow$ ar (ABD) = $\frac{1}{2}$ ar (ABC) ... (i)

Now, in ΔABD, BE is the median and median divides a triangle into two triangles of equal area.
So, ar (BED) = ar (ABE) ... (ii)

$\Rightarrow$ ar (BED) = $\frac{1}{2}$ ar(ABD) (Since, ar(BAE) = $\frac{1}{2}$ ar(ABD))

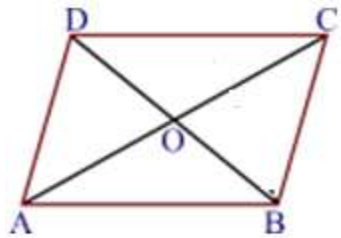
$\Rightarrow$ ar (BED) = $\frac{1}{2} \times \frac{1}{2}$ ar (ABC)   (From eq. (i))

$\Rightarrow$ Thus, ar (BED) = $\frac{1}{4}$ ar (ABC)

Hence Proved.

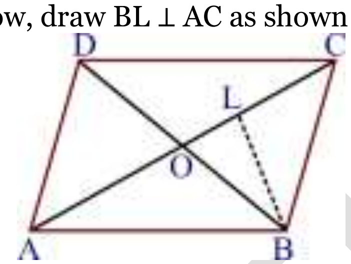
Q.3 Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Sol. **Given:** Let ABCD be a parallelogram and AC and BD are diagonals.



To prove: Diagonals AC and BD divide the parallelogram ABCD into four triangles of equal area.

Construction: Now, draw BL $\perp$ AC as shown in figure.



Proof: Since, ABCD is a parallelogram and so diagonals AC and BD bisect each other at O.

So, AO = OC and BO = OD...... (i)

Now, ar (AOB) = $\frac{1}{2}$ × AO × BL

ar (OBC) = $\frac{1}{2}$ × OC × BL

Since, AO = OC (from eq. (i))

So, ar (AOB) = ar (OBC).........(ii)

In the same way,

ar (OBC) = ar (OCD) ; ar (OCD) = ar(ODA) ;

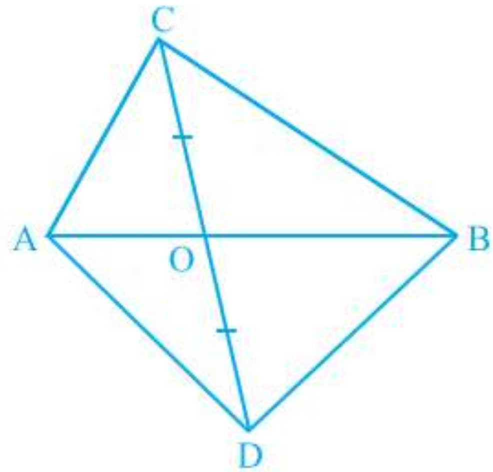
ar (ODA) = ar (OAB) ; ar (OAB) = ar (OBC);

ar (OCD) = ar (ODA)

Hence , ar (OAB) = ar (OBC) = ar(OCD) = ar (OAD)

Hence Proved.

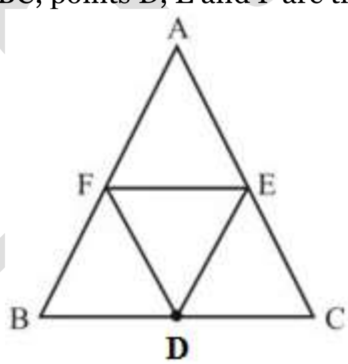
Q.4 In figure, ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Sol. **Given:** ABC and ABD are two triangles on the same base AB. A line segment AB bisects the line CD at point O. So, OC = OD.
To prove: ar (ABC) = ar (ABD).
Proof: Firstly, In Δ ACD,
OC = OD (Given)
So, AO is the median of triangle ACD.
Since, median divides a triangle in two triangles of equal area.
Therefore, ar (AOC) = ar (AOD) ....(i)
In the same way, BO is the median of triangle BCD.
So, ar (BOC) = ar (BOD) ....(ii)
By adding (i) & (ii),
ar (AOC) + ar (BOC) = ar (AOD) + ar (BOD)
⇒ Thus, ar (ABC) = ar (ABD)
Hence Proved.

Q.5 D, E and F are respectively the mid- points of the side BC, CA and AB of a ΔABC. Show that
(i) BDEF is a parallelogram
(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)
(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC).

Sol. **Given:** In a Δ ABC, points D, E and F are the mid- points of the sides BC, CA and AB respectively.



To prove: (i) BDEF is a parallelogram.
(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)
(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

(i) Proof: In ΔABC,
Since, points E and F are the mid- points of sides AC and AB respectively.
So, EF || BC (From the mid- point theorem)
Therefore EF || BD... (i)
Since, points E and D are the mid- points sides AC and BC respectively.
So, ED || AB (From the mid- point theorem)
Therefore ED || BF... (ii)
From (i) & (ii),
Thus, BDEF is a parallelogram.
Hence Proved.

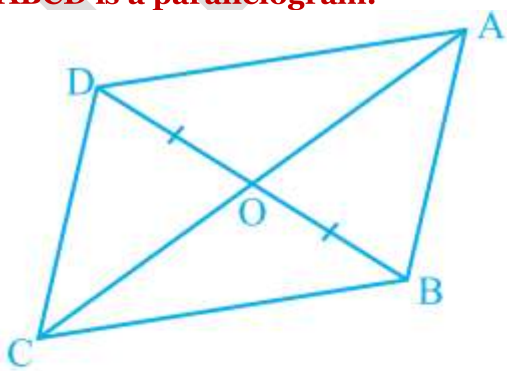
(ii)Proof: As the above, quadrilaterals FDCE and AFDE are parallelogram.
Since, FD is a diagonal of parallelogram BDEF.
So, ar (FBD) = ar (DEF) ... (iii)
Since, ED is a diagonal of parallelogram FDCE.
ar (DEC) = ar (DEF) ... (iv)
Since, FE is a diagonal of parallelogram AFDE.
So, ar (AFE) = ar (DEF) ... (v)
From (iii), (iv) & (v)
ar (FBD) = ar (DEC) = ar (AFE) = ar (DEF) .... (vi)
ar (ABC) = ar (AFE) + ar (FBD) + ar (DEC) + ar (DEF)
from eq. (vi)
ar(ABC) = 4 ar(DEF)
$\Rightarrow$ ar (DEF) = $\frac{1}{4}$ ar (ABC).

(iii) Proof: And also, ar (BDEF) = 2 ar (DEF)

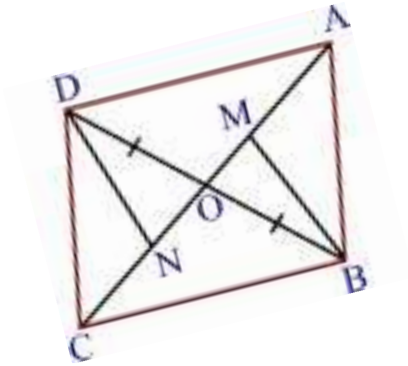
$= 2 \times \frac{1}{4}$ ar (ABC)

$= \frac{1}{2}$ ar (ABC)............Hence proved.

Q.6 In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:
(i) ar (DOC) = ar (AOB)
(ii) ar (DCB) = ar (ACB)
(iii) DA || CB or ABCD is a parallelogram.



***Sol.* Given:** A quadrilateral ABCD in which diagonals AC and BD intersect at O such that OB = OD and AB = CD.
To prove: (i) ar (DOC) = ar (AOB)
(ii) ar (DCB) = ar (ACB)
(iii) DA || CB or ABCD is a parallelogram.
Construction: Draw DN $\perp$ AC and BM $\perp$ AC.



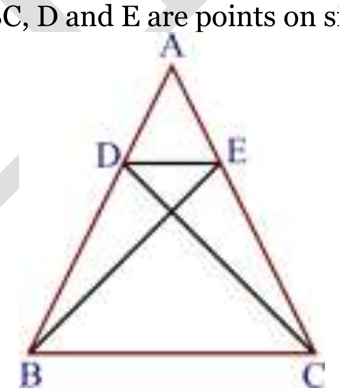
(i) Proof: Firstly, in ΔDON and ΔBOM
∠DNO = ∠BMO (Since, DN ⊥ AC and BM ⊥ AC)
∠DON = ∠BOM [Vertical opposite angles)
OD = OB (Given)
So, from AAS criterion of congruent.
ΔDON ≅ ΔBOM ... (i)
Now, in ΔDCN and ΔBAM
∠DNC = ∠BMA (Since, DN ⊥ AC and BM ⊥ AC)
DC = AB (Given)
DN = BM (Since, ΔDON≅ΔBOM⇒ So, by CPCT; DN=BM)
So, from RHS criterion of congruence,
ΔDCN ≅ ΔBAM ... (ii)
From eq. (i) & (ii),
ar (DON) + ar (DCN) = ar (BOM) + ar (BAM)
⇒Thus, ar (DOC) = ar (AOB)...............Hence Proved.

(ii) Since, we have proved that ar (DOC) = ar (AOB)
By adding ar (BOC) in both sides,
So, ar (DOC) + ar (BOC) = ar (AOB) + ar (BOC)
⇒Thus, ar (DCB) = ar (ACB).............Hence Proved.

(iii) Since, ΔDCB and ACB have equal areas and lie on the same base. So, these triangles lie between the same parallels.
⇒ DA || CB
Therefore, ABCD is a parallelogram.

Q.7 D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE|| BC.

Sol. **Given:** In a ΔABC, D and E are points on sides AB and AC respectively such that ar (DBC) = ar (EBC)

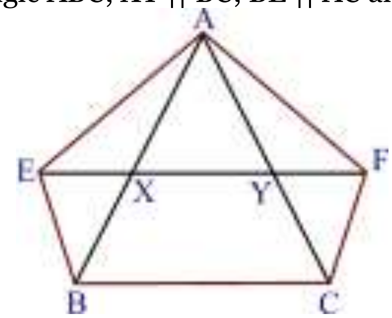


To Prove: DE|| BC
Proof: Since, ΔDBC and ΔEBC have equal area and have a same base BC.
So, altitude from D of ΔDBC = Altitude from E of ΔEBC.
⇒ Therefore, ΔDBC and ΔEBC are in between the same parallel lines.
⇒ Thus, DE || BC.

Hence Proved.

Q.8 XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar (ABE) = ar (ACF).

***Sol.* Given:** In a triangle ABC, XY || BC, BE || AC and CF || AB.



To Prove: ar (ABE) = ar (ACF)

Proof: Since, given that XY || BC and BE || CY

Therefore, BCYE is a parallelogram.

Since, ΔABE and parallelogram BCYE are on the same base BE and in between the same parallel lines BE and AC.

So, ar(ABE) = $\frac{1}{2}$ ar(BCYE) ... (i)

Now, given that CF || AB and XY || BC

⇒Therefore, BCFX is a || gm

Since, ΔACF and parallelogram BCFX are on the same base CF and in between the same parallel AB and FC .

So, ar (ACF) = $\frac{1}{2}$ ar (BCFX) ... (ii)

But from the figure, parallelogram BCFX and parallelogram BCYE are on the same base BC and in between the same parallels BC and EF.

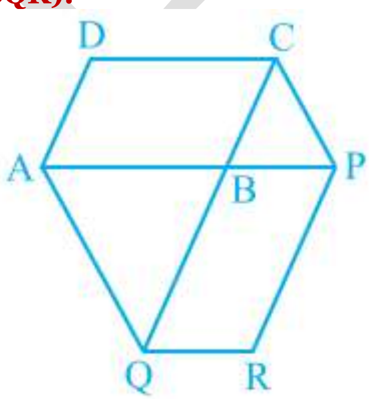
So, ar (BCFX) = ar (BCYE) ... (iii)

Therefore, from eq. (i), (ii) and (iii), we can write

ar (ΔABE) = ar (ΔACF)

Hence Proved.

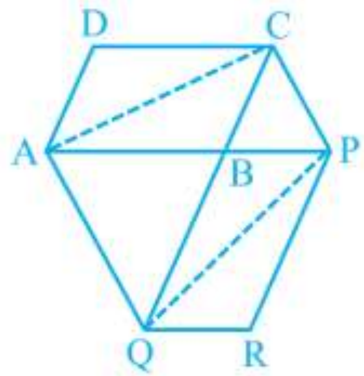
Q.9 The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see figure). Show that ar(ABCD) = ar (PBQR).



***Sol.* Given:** A parallelogram ABCD, AQ || CB and Parallelogram PBQR.

To Prove: ar (ABCD) = ar (PBQR)

Construction: Join AC and PQ.



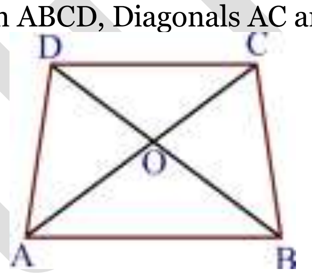
Proof: Since, AC and PQ are diagonals of parallelogram ABCD and parallelogram BPQR respectively.

So, ar (ABC) = $\frac{1}{2}$ ar (ABCD) ... (i)

and ar (PBQ) = $\frac{1}{2}$ ar (BPRQ) ... (ii)

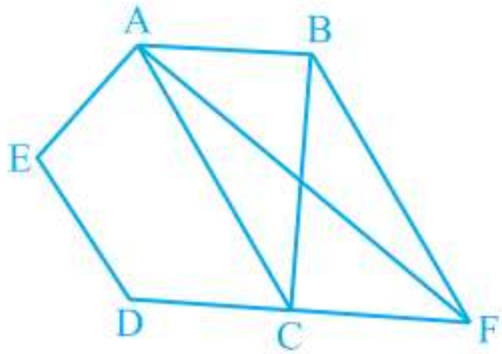
Now from the figure, ΔACQ and ΔAQP lie on the same base AQ and in between the same parallels AQ and CP.
So, ar(ACQ) = ar(AQP)
By, subtracting ar(ABQ) from both sides,
⇒ ar (ACQ) – ar (ABQ) = ar (AQP) – ar (ABQ)
⇒ ar (ABC) = ar (BPQ)
From eq. (i) & (ii),
⇒ $\frac{1}{2}$ ar(ABCD) = $\frac{1}{2}$ ar(BPRQ)
⇒ ar(ABCD) = ar (BPRQ)............Hence Proved.

Q.10 Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).
***Sol.* Given:** In a trapezium ABCD, Diagonals AC and BD intersect each other at O and AB || DC.



To prove: ar (AOD) = ar (BOC).
Proof: Since, ΔABC and ΔABD lie on the same base AB and in between the same parallel lines AB and DC.
So, ar(ABD) = ar(ABC)
By subtracting ar (AOB) from both sides,
⇒ ar(ABD) – ar(AOB) = ar(ABC) – ar (AOB)
⇒ Thus, ar (AOD) = ar(BOC)
Hence Proved.

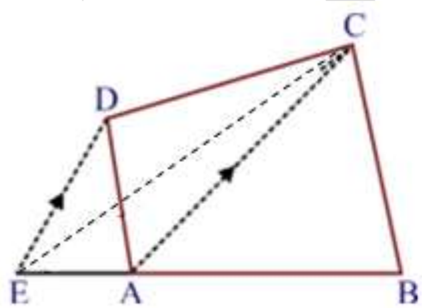
Q.11 In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that
(i) ar (ACB) = ar (ACF)
(ii) ar (AEDF) = ar (ABCDE)



***Sol.* Given:** In a pentagon ABCDE,BF || AC
To prove: (i) ar (ACB) = ar (ACF)
(ii) ar (AEDF) = ar (ABCDE)
(i) Proof: From the figure, ΔACB and ΔACF lie on the same base AC and in between the same parallel lines AC and BF.
So, ar (ACB) = ar (ACF).............Hence Proved.
(ii) Proof: Since, we have proved (ACB) = ar (ACF)
By adding ar(ACDE) on both sides,
ar (ACF) + ar (ACDE) = ar (ACB) + ar (ACDE)
⇒ Thus, ar (AEDF) = ar (ABCDE)
Hence Proved.

Q.12 A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Sol. Let ABCD be the quadrilateral plot of land. Now, produce BA to meet CD drawn parallel to CA at E i.e. ED || AC. Join EC.

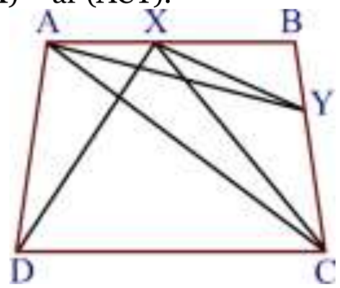


From the figure, ΔEAC and ΔADC are on the same parallel lines DE and CA
So, ar (EAC) = ar (ADC).......... (i)
Now ar (ABCD) = ar (ABC) + ar (ACD)
= ar (ABC) + ar (ADC)
= ar (ABC) + ar (EAC)
= ar (EBC)
It means quad. ABCD = Δ EBC
Thus, this is required explain to the suggested proposal.

Q.13 ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

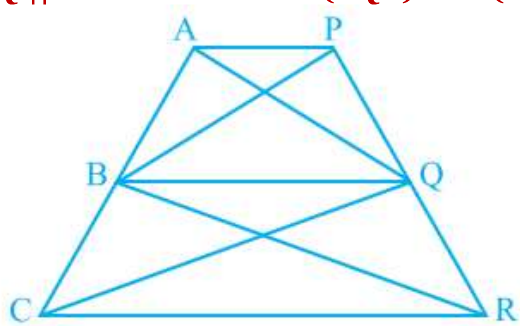
***Sol.* Given:** A trapezium ABCD with AB || DC and XY || AC.

To prove: ar (ADX) = ar (ACY)
Construction: ar (ADX) = ar (ACY).



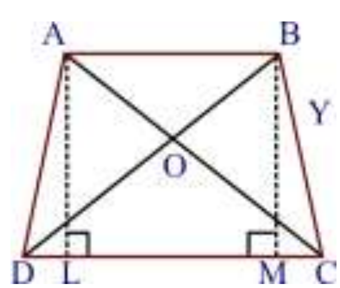
Proof: Since, from the figure ΔACX and ΔACY are on the same base AC and in between same parallel lines AC and XY**.**
So, ar (ACX) = ar (ACY) ... (i)
Again from the figure, ΔACX and ΔADX are on same base AX and in between same parallel lines AB and DC.
So, ar (ACX) = ar (ADX) ...... (ii)
From (i) & (ii),
Thus, ar(ADX) = ar (ACY)...............Hence Proved.

Q.14 In figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR)



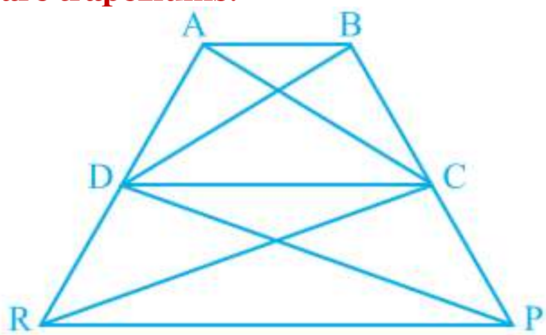
***Sol.* Given:** AP || BQ || CR
To prove: ar (AQC) = ar (PBR)
Proof: From the figure,
ar (AQC) = ar (AQB) + ar (BQC) ... (i)
and ar (PBR) = ar (PBQ) + ar (QBR) ... (ii)
Since, ΔAQB and ΔPBQ are on the same base BQ and in between same parallel lines AP and BQ.
So, ar (AQB) = ar(PBQ) ....(iii)
Since, ΔBQC and ΔQBR are on the same base BQ and in between same parallel lines BQ and CR.
Also, ar (BQC) = ar (QBR)...............(iv)
By using (iii) and (iv) in (i) and (ii),:
Thus, ar (AQC) = ar (PBR).........Hence Proved.

Q.15 Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.
***Sol.* Given:** In a quadrilateral ABCD, **d**iagonals AC and BD intersect at O in such a way that ar (AOD) = ar (BOC)



To prove: ABCD is a trapezium.
Construction: Draw the altitude AL and BM on side BC.
Proof: Since, ar (AOD) = ar (BOC) .... (i)
By adding ar(ODC) on both sides,
ar(AOD) + ar (ODC) = ar (BOC) + ar (ODC)
⇒ ar(ADC) = ar (BDC)
$\Rightarrow \frac{1}{2} \times DC \times AL = \frac{1}{2} \times DC \times BM$
⇒ AL = BM and AB || DC
Thus, ABCD is a trapezium.
Hence Proved.

Q.16 In figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



***Sol.* Given:** In given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC)
To prove: quadrilaterals ABCD and DCPR are trapeziums.
Proof: Since, from the figure
ar (BDP) = ar (ARC) and ar (DPC) = ar (DRC) [Given]
By subtracting both,
ar (BDP) – ar (DPC) = ar (ARC) – ar (DRC)
⇒ ar (BDC) = ar (ADC)
Since, ΔBDC and ΔADC have equal areas and have the same base. So, these triangles lie between the same parallel lines.
⇒So, DC || AB
Thus, ABCD is a trapezium...............Hence Proved.
Since, from the figure,
ar (DRC) = ar (DPC) [Given]
BY subtracting ar (DLC) from both sides,
ar (DRC) – ar (DLC) = ar (DPC) – ar (DLC)
⇒ ar (DLR) = ar (CLP)
By adding ar (RLP) to both sides,
ar (DLR) + ar(RLP) = ar(CLP) + ar(RLP)
⇒ ar (DRP) = ar (CRP)
Since, Δ DRP and Δ CRP have equal areas and have the same base. So, these triangles lie between the same parallel lines.
⇒So, RP || DC
Thus, DCPR is a trapezium................Hence Proved.