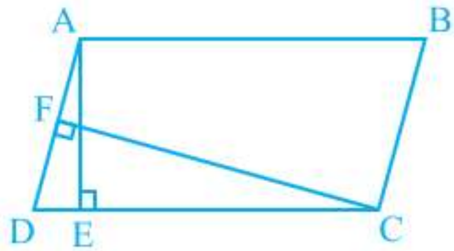
Areas of Parallelogram and Triangles: Exercise 9.2

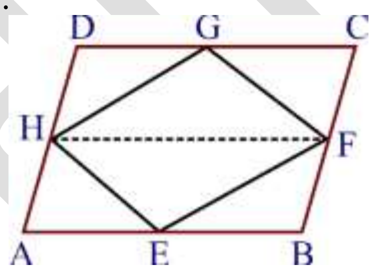
Q.1 In figure ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Sol. Given: ABCD is a parallelogram in which AE ⊥ DC & CF ⊥ AD and AB = 16 cm, AE = 8 cm and CF = 10 cm.
So, Area of || gm ABCD = AB×AE (Since, Area of a ||gm = Base × Height)
$= (16 \times 8)\ cm^2$
$= 128 cm^2$ ...(i)
And also area of ||gm ABCD = AD × CF
$= (AD \times 10)\ cm^2$ ... (ii)
Since, both the area of the same parallelogram ABCD,
So, from (i) and (ii),
128 = AD × 10
⇒ Thus, AD = $\frac{128}{10}$
= 12.8cm

Q.2 If E, F, G and H are respectively the mid - points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Sol. Given: A parallelogram ABCD in which E, F, G and H are respectively the mid - points of the sides AB, BC, CD and DA respectively.



Construction: Join HF.
To Prove: ar (EFGH) = ar (ABCD)
Proof: Since, Δ HGF and parallelogram HDCF are on the same base HF and in between the same parallel lines HF and DC.

So, ar(ΔHGF) = $\frac{1}{2}$ ar(HDCF) ... (i)

In the same way, ΔHEF and parallelogram ABFH are on the same base HF and in between the same parallel lines HF and AB.

So, ar(ΔHEF) = $\frac{1}{2}$ ar (ABFH) ... (ii)

So, by adding (i) & (ii),

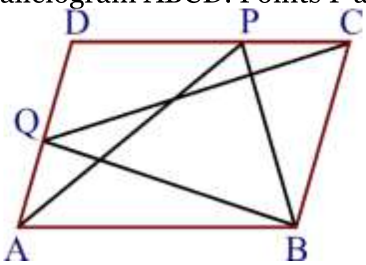
$\text{ar}(\Delta HGF) + \text{ar}(\Delta HEF) = \frac{1}{2}\text{ar}(HDCF) + \frac{1}{2}\text{ar}(ABFH)$

$\Rightarrow$Thus, $\text{ar}(EFGH) = \frac{1}{2}\text{ar}(ABCD)$

Hence Proved.

Q.3 P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar(APB) = ar(BQC).

Sol. Given: A parallelogram ABCD. Points P and Q are the two points on the sides DC and AD respectively.



To Prove: ar(APB) = ar(BQC)

Construction: Join QB, QC, PA and PB in figure.

Proof: In given figure. ΔAPB and parallelogram ABCD are on the same base AB and in between the same parallel lines AB and DC.

So, $\text{ar}(\Delta APB) = \frac{1}{2}\text{ar}(ABCD)$.... (i)

In the same way, ΔBQC and parallelogram ABCD are on the same base BC and in between the same parallel lines BC and AD.

So, $\text{ar}(\Delta BQC) = \frac{1}{2}\text{ar}(ABCD)$ .... (ii)

From (i) & (ii),

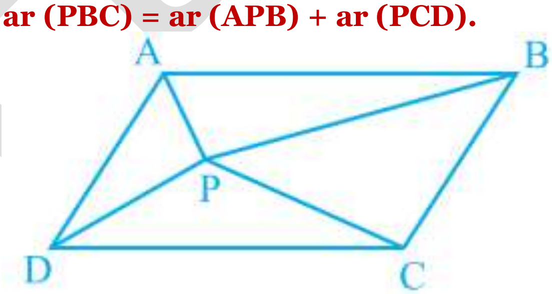
Thus, ar(ΔAPB) = ar (ΔBQC)

Hence Proved.

Q.4 In figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD).

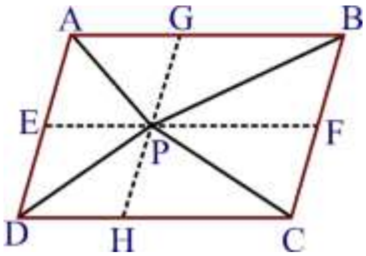


***Sol.* Given:** A parallelogram ABCD. Point P is an interior point.

To prove: (i) ar (APB) + ar (PCD) = ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD).

Construction: Draw line EPF parallel to AB or DC and similarly draw GPH parallel to AD or BC.



Now, quadrilateral AGHD is parallelogram. (Since GH || DA and AG|| DH)
Similarly, quadrilaterals HCBG, EFCD and ABFE are parallelograms.

(i) Proof: In given figure, ΔAPB and parallelogram ABFE are on the same base AB and in between the same parallel lines AB and EF.

So, $\text{ar}(APB) = \frac{1}{2}\text{ar}(ABFE)$ ... (i)

Similarly, $\text{ar}(PCD) = \frac{1}{2}\text{ar}(EFCD)$ ... (ii)

By adding (i) & (ii),

$\text{ar}(APB) + \text{ar}(PCD) = \frac{1}{2}[\text{ar}(ABFE) + \text{ar}(EFCD)]$

Thus, $\text{ar}(APB) + \text{ar}(PCD) = \frac{1}{2}\text{ar}(ABCD)$....(iii)

Hence Proved.

(ii) Proof: In given figure, ΔAPD and parallelogram AGHD are on the same base AD and in between the same parallel lines AD and HG.

So, $\text{ar}(APD) = \frac{1}{2}\text{ar}(AGHD)$....(iv)

In the same way, $\text{ar}(PCB) = \frac{1}{2}\text{ar}(GBCH)$....(v)

By adding (iv) & (v),

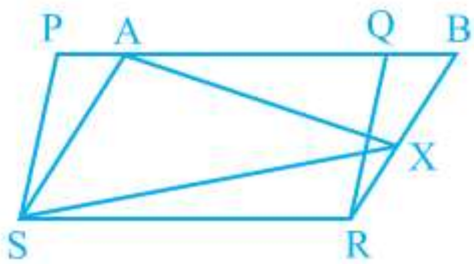
$\text{ar}(APD) + \text{ar}(PCB) = \frac{1}{2}[\text{ar}(AGHD) + \text{ar}(GBCH)]$

$\text{ar}(APD) + \text{ar}(PCB) = \frac{1}{2}\text{ar}(ABCD)$...(vi)

From (iii) & (vi),
Thus, ar (APD) + ar (PBC) = ar (APB) + ar(PCD)
Hence Proved.

Q.5 In figure PQRS and ABRS are parallelograms and X is any point on side BR. Show that
(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS).



Sol. **Given:** PQRS and ABRS are parallelograms and X is any point on side BR.
To prove: (i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ ar (PQRS)

(i) Proof: In given figure, parallelogram PQRS and parallelogram ABRS are on the same base RS and in between the same parallel lines SR and PAQB.
So, ar (PQRS) = ar (ABRS) ... (i)
Hence Proved.

(ii)Proof: In given figure, ΔAXS and parallelogram ABRS are on the same base AS and in between the same parallel lines AS and RB.

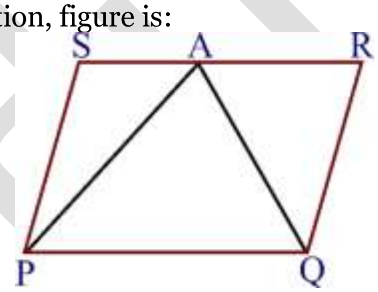
So, ar (AXS) = $\frac{1}{2}$ ar (ABRS)

⇒ ar (AXS) = $\frac{1}{2}$ ar (PQRS)   (From eq. (i))

Hence Proved.

Q.6 A farmer was having a field in the form of a parallelogram PQRS. He took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should he do it?

Sol. According to question, figure is:



From the figure, parallelogram PQRS is divided into three parts. Each part is of the shape of triangle.
Since, in figure ΔAPQ and parallelogram PQRS are on the same base PQ and in between the same parallel lines PQ and SR.

So, ar (APQ) = $\frac{1}{2}$ ar (PQRS) ... (i)

Now, ar (APS) + ar (AQR) = ar (PQRS) − ar (APQ)

= ar (PQRS) − $\frac{1}{2}$ ar(PQRS)   (From eq.(1))

= $\frac{1}{2}$ ar (PQRS)....(ii)

So, from (i) & (ii),

ar (APS) + ar (AQR) = ar (APQ)
Therefore, the farmer should sow wheat and pulses either as:
Wheat-(ΔAPS and ΔAQR)
Pulses- (Δ APQ)
or
Wheat- (ΔAPQ)
Pulses- (ΔAPS and ΔAQR)