Areas Related to Circles: Exercise 12.2

Q.1 Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol. Since, area A of a sector of angle θ in a circle of radius r is,

$A = (\theta/360) \times \pi r^2$

given: r = 6 cm and θ = 60°

Thus, $A = [\frac{60}{360} \times \frac{22}{7} \times 36]\ cm^2$

$= (132/7)\ cm^2$

Q.2 Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Given: Circumference = 22 cm

Suppose, r is the radius of the circle.

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2 \times (22/7) \times r = 22$

$\Rightarrow r = 7/2 cm$

Area of the quadrant of a circle = $(1/4)\ \pi r^2$

$= (\frac{1}{4} \times \frac{22}{7} \times \frac{49}{4})\ cm^2$

$= \frac{539}{56}\ cm^2 = 77/8\ cm^2$

Q.3 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. Since, minute hand of a clock rotates in a circle of radius equal to its length i.e., 14 cm. As we know that the minutes hand rotates through 6° in one minute, so, area swept by the minute hand in 5 minutes.

$\text{Area} = (\frac{\theta}{360} \times \pi r^2 \times 5)\ cm^2$

$= [\frac{6}{360} \times \frac{22}{7} \times (14)^2 \times 5]\ cm^2$

$= (\frac{1}{60} \times \frac{22}{7} \times 196 \times 5)\ cm^2$

$= 154/3\ cm^2$

Q.4 A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment (ii) major sector (Use π = 3.14)

Sol. Given: r = 10 cm, θ = 90°

(i) Minor segment:

$\text{Area} = r^2 [\frac{\pi\theta}{360} - \frac{1}{2}\sin\theta]$

$= (10)^2 [\frac{3.14 \times 90}{360} - \frac{1}{2}\sin 90^\circ]\ cm^2$

$= 100\ (0.785 - 0.5)\ cm^2$

$= (100 \times 0.285)\ cm^2$

$= 28.5 cm^2$

(ii) Major sector:

$\text{Area} = \frac{\theta}{360} \times \pi r^2$, where θ = 270°

$= (\frac{270}{360} \times 3.14 \times 10^2)\ cm^2$

$= (\frac{3}{4} \times 3.14 \times 100)\ cm^2$

$= 235.5 cm^2$

Q.5 In a circle of a radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc
(ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord.

Sol. Given: Radius, r = 21 cm and Angle subtended, θ=60°

(i) Length of the arc: $\ell = \frac{\theta}{180} \times \pi r$

$= (\frac{60}{180} \times \frac{22}{7} \times 21) cm$

$= 22 cm$

(ii) Area of the sector: Area $= \frac{\theta}{360} \times \pi r^2$

$= (\frac{60}{360} \times \frac{22}{7} \times 21 \times 21)\ cm^2$

$= 231 cm^2$

(iii) Area of the segment: Area $= r^2 [\frac{\pi\theta}{360} - \frac{1}{2}\sin\theta]$

$= (21)^2 [\frac{22}{7} \times \frac{60}{360} - \frac{1}{2}\sin 60^\circ]\ cm^2$

$= 441(\frac{11}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2})\ cm^2$

$= (21 \times 11 - \frac{441\sqrt{3}}{4})\ cm^2$

$= (231 - \frac{441\sqrt{3}}{4})\ cm^2$

Q.6 A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segment of the circle. (Use π = 3.14 and √3 =1.73)

Sol. Given: Radius, r = 15 cm and θ = 60°

So, area of the minor segment $= r^2 [\frac{\pi\theta}{360} - \frac{1}{2}\sin\theta]$

$= (15)^2[\frac{314 \times 60}{360} - \frac{1}{2} \times \sin 60]\ cm^2$

$= 225 [\frac{314}{6} - \frac{1}{2} \times \frac{\sqrt{3}}{2}]\ cm^2$

$= 225(\frac{6.28 - 3x1.73}{12})\ cm^2$

$= \frac{225 \times 1.09}{12} cm^2$

$= 20.4375 cm^2$

Area of the major segment = Area of the circle – Area of the minor segment

$= (3.14 \times 225 - 20.4375)\ cm^2$

$= (706.5 - 20.4375)\ cm^2$

$= 686.0625\ cm^2$

Q.7 A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle.
(Use π = 3.14 and $\sqrt{3}$ =1.73)

Sol. Given: Radius, r = 12 cm θ = 120°

Area of the corresponding segment of the circle = Area of the minor segment

$$= r^2 \left[\frac{\pi\theta}{360} - \frac{1}{2}\sin\theta\right]$$

$$= (12)^2\left[\frac{3.14\times120}{360} - \frac{1}{2}\sin120^\circ\right] \text{ cm}^2$$

since, sin120° = sin(180° – 60°) = sin60° = 3√2

$$= 144\left(\frac{3.14}{3} - \frac{1}{2} \text{x} \frac{\sqrt{3}}{2}\right) \text{ cm}^2$$

$$= 144 \times \frac{3.14\text{x}4 - 3\sqrt{3}}{12} cm^2$$

$$=12 \times (12.56 - 3\times1.73) \text{ cm}^2$$

$$=12 \times (12.56 - 5.19) \text{ cm}^2$$

$$=12\times7.37\text{cm}^2 = 88.44 \text{ cm}^2$$

Q.8 A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see figure). Find

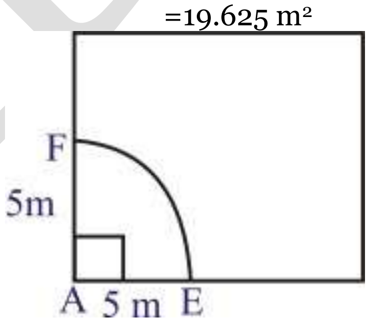
(i) The area of that part of the field in which the horse can graze.
(ii) The increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

***Sol.* (i)** Since, horse will graze over a quadrant of a circle with centre at the corner of the square field and radius = 5 m.

So, the area of the quadrant of this circle = $\frac{\pi\times5^2}{4} = \frac{3.14\times25}{4}$ m²

$$= 78.5/4$$

$$=19.625 \text{ m}^2$$

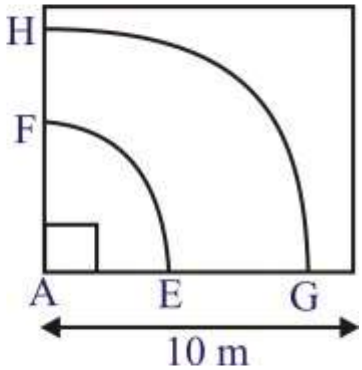


(ii) In the 2nd case, increase in the grazing area if the rope were 10 m long i.e. radius = 10 m.

The area of the quadrant of this circle = $\frac{\pi\times10^2}{4}$ m²

$$= \frac{3.14\times100}{4} \text{ m}^2$$

$$=78.5\text{m}^2$$



Thus, increase in the grazing area = $(78.5 - 19.625)\ m^2$
$= 58.875 m^2$

Q.9 A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:

(i) The total length of the silver wire required.
(ii) The area of each sector of the brooch.

***Sol.* (i)** Silver wire used to make a brooch:

$$= 2\pi r, \text{ where } r = \frac{35}{2} \text{ mm}$$

$$= 2 \times \frac{22}{7} \times \frac{35}{2} \text{ mm}$$

$$= 110 \text{mm}$$

Wire used in 5 diameters = 5×35 mm = 175 mm
So, total wire used = $(110 + 175)$ mm = 285 mm

(ii) The area of each sector of the brooch $= \frac{1}{10} \times$ Area of the circle

$$= \frac{1}{10} \times \pi r^2, \text{ where } r = 35/2 \text{ mm}$$

$$= \left(\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) \text{mm}^2$$

$$= 385/4 \text{ mm}^2$$

Q.10 An umbrella has 8 ribs which are equally spaced (see figure). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Sol. Given: Radius of circle= 45 cm

Area between two consecutive ribs = $\frac{1}{8} \times \pi r^2$,

$$= (\frac{1}{8} \times \frac{22}{7} \times 45 \times 45) \text{ cm}^2$$
$$= 22275/28 \text{ cm}^2$$

Q.11 A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol. Given: radius of wiper, r = 25 cm, θ =115°

So, total area cleaned at each sweep of the blades = 2 × Area of the sector = $2 \times \frac{\theta}{360} \times \pi r^2$

$$= (2 \times \frac{115}{360} \times \frac{22}{7} \times 25 \times 25) \text{ cm}^2$$
$$= 158125/126 \text{ cm}^2$$

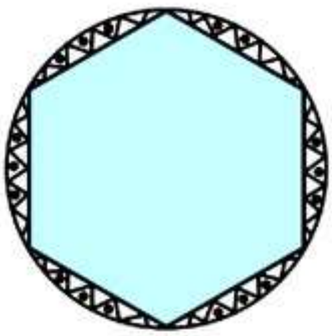
Q.12 To warn ships for underwater rocks a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π=3.14)

Sol .Given: (Radius r = 16.5 km and θ = 80°)

Area of the sea over which the ships are warned = Area of the sector

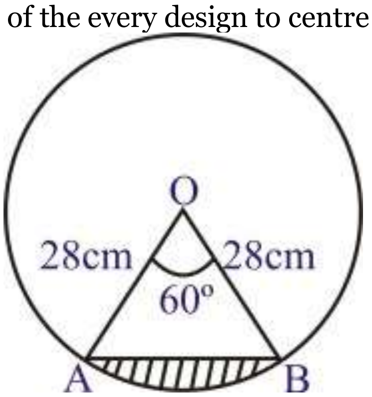
$$= \frac{\theta}{360} \times \pi r^2$$
$$= (\frac{80}{360} \times 3.14 \times 16.5 \times 16.5) \text{ sq.km}$$
$$= 68389.2/360 \text{sq.km}$$
$$= 189.97 \text{ km}^2$$

Q.13 A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm². (Use √3=1.7)



Sol. Given: radius r = 28 cm

Firstly, we join the corner of the every design to centre of the circle.



So, area of one design = Area of the sector AOB – Area (ΔAOB)

$= \left(\frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r \times r \times \sin 60^\circ\right) \text{ cm}^2$

$= \left(\frac{60}{360} \times \frac{22}{7} \times 28 \times 28 - \frac{1}{2} \times 28 \times 28 \times \frac{\sqrt{3}}{2}\right) \text{ cm}^2$

$= \left(\frac{1232}{3} - 333.2\right) \text{ cm}^2$

$= \left(\frac{1232 - 999.6}{3}\right) \text{ cm}^2$

$= \frac{232.4}{3} \text{ cm}^2$

Therefore, area of 6 designs = $\left(6 \times \frac{232.4}{3}\right) \text{ cm}^2 = 464.8 \text{ cm}^2$

According to question, cost of making such designs @ Rs. 0.35 per cm^2

= Rs (0.35 × 464.8)

= Rs. 162.68

Q.14 Tick the correct answer in the following: Area of a sector of angle p (in degrees) of a circle with radius R is:

(a) $\frac{p}{180} \times 2\pi R$ **(b)** $\frac{p}{180} \times \pi R^2$

(c) $\frac{p}{360} \times 2\pi R$ **(d)** $\frac{p}{720} \times 2\pi R^2$

Sol. Since, area A of a sector of angle θ in a circle of radius r is given by

$A = \frac{\theta}{360} \times \pi r^2$

Given: Radius, r = R and θ = p

Thus, Area, $A = \frac{p}{360} \times \pi R^2 = \frac{p}{720} \times 2\pi R^2$

Thus, Correct option: (D)