Algebraic Expressions and Identities: Exercise 9.5

Q.1 Use a suitable identity to get each of the following products.

(i) $(x + 3)(x + 3)$ **(ii) $(2y + 5)(2y + 5)$** **(iii) $(2a - 7)(2a - 7)$**

(iv) $(3a - \frac{1}{2})(3a - \frac{1}{2})$ **(v) $(1.1m - 0.4)(1.1m + 0.4)$** **(vi) $(a^2 + b^2)(-a^2 + b^2)$**

(vii) $(6x - 7)(6x + 7)$ **(viii) $(-a + c)(-a + c)$** **(ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$**

(x) $(7a - 9b)(7a - 9b)$

Sol. **(i) Given:** $(x + 3)(x + 3) = (x + 3)^2$

$= (x)^2 + 2 \times x \times 3 + (3)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= x^2 + 6x + 9$

(ii) Given: $(2y + 5)(2y + 5) = (2y + 5)^2$

$= (2y)^2 + 2 \times 2 \times y \times 5 + (5)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 4y^2 + 20y + 25$

(iii) Given: $(2a - 7)(2a - 7) = (2a - 7)^2$

$= (2a)^2 - 2 \times 2a \times 7 + (7)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 4a^2 - 28a + 49$

(iv) Given: $(3a - \frac{1}{2})(3a - \frac{1}{2}) = \left(3a - \frac{1}{2}\right)^2$

$= (3a)^2 - 2 \times 3a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 9a^2 - 3a + \frac{1}{4}$

(v) Given: $(1.1m - 0.4)(1.1m + 0.4) = (1.1m)^2 - (0.4)^2$

$= 1.21m^2 - 0.16$ (Since, $(a - b)(a + b) = a^2 - b^2$)

(vi) Given: $(a^2 + b^2)(-a^2 + b^2) = (b^2)^2 - (a^2)^2$ (Since, $(a - b)(a + b) = a^2 - b^2$)

$= b^4 - a^4$

(vii) Given: $(6x - 7)(6x + 7) = (6x)^2 - (7)^2$ (Since, $(a - b)(a + b) = a^2 - b^2$)

$= 36x^2 - 49$

(viii) Given: $(-a + c)(-a + c) = (c - a)(c - a)$

$= (c - a)^2$

$= (c)^2 - 2\,c\,a + (a)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= c^2 - 2ca + a^2$

(ix) Given: $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right) = \left(\frac{x}{2} + \frac{3y}{4}\right)^2$

$= \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{3y}{4} + \left(\frac{3y}{4}\right)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= \frac{x^2}{4} + \frac{3}{4}xy + \left(\frac{9}{16}\right)y^2$

(x) Given: $(7a - 9b)(7a - 9b) = (7a - 9b)^2$

$= (7a)^2 - 2 \times 7a \times 9b + (9b)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 49a^2 - 126ab + 81b^2$

Q.2 Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.

(i) $(x + 3)(x + 7)$ **(ii) $(4x + 5)(4x + 1)$**

(iii) $(4x - 5)(4x - 1)$ **(iv) $(4x + 5)(4x - 1)$**

(v) $(2x + 5y)(2x + 3y)$ **(vi) $(2a^2 + 9)(2a^2 + 5)$**

(vii) $(xyz - 4)(xyz - 2)$

Sol. Product of given expressions:

(i) Given: $(x + 3)(x + 7) = (x)^2 + (3+7)x + 3\times7$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= x^2 + 10x + 21$

(ii) Given: $(4x + 5)(4x + 1) = (4x)^2 + (5 + 1)4x + 5\times1$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= 16x^2 + 24x + 5$

(iii) Given: $(4x - 5)(4x - 1) = (4x)^2 + (-5 - 1)4x + (-5)\times(-1)$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= 16x^2 - 24x + 5$

(iv) Given: $(4x + 5)(4x - 1) = (4x)^2 + (5 - 1)4x + (5)\times(-1)$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= 16x^2 + 16x - 5$

(v) Given: $(2x + 5y)(2x + 3y) = (2x)^2 + (5y + 3y)2x + 5y\times3y$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= 4x^2 + 16xy + 15y^2$

(vi) Given: $(2a^2 + 9)(2a^2 + 5) = (2a^2)^2 + (9 + 5)\,2a^2 + 9 \times 5$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= 4a^4 + 28a^2 + 45$

(vii) Given: $(xyz - 4)(xyz - 2) = (xyz)^2 + (-4-2)\,xyz + (-4)\times(-2)$

(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= x^2y^2z^2 - 6xyz + 8$

Q.3 Find the following squares by using the identities.

(i) $(b - 7)^2$ **(ii) $(xy + 3z)^2$** **(iii) $(6x^2 - 5y)^2$**

(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$ **(v) $(0.4p - 0.5q)^2$** **(vi) $(2xy + 5y)^2$**

***Sol.* (i) Given:** $(b - 7)^2 = (b)^2 - 2\times b\times7 + (7)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= b^2 - 14b + 49$

(ii) Given: $(xy + 3z)^2 = (xy)^2 + 2\times xy\times3z + (3z)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= x^2y^2 + 6xyz + 9z^2$

(iii) Given: $(6x^2 - 5y)^2 = (6x^2)^2 - 2\times6x^2\times5y + (5y)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 36x^4 - 60x^2y + 25y^2$

(iv) Given: $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2 = \left(\frac{2}{3}m\right)^2 + 2\times\frac{2}{3}m\times\frac{3}{2}n + \left(\frac{3}{2}n\right)^2$

(Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$

(v) Given: $(0.4p - 0.5q)^2 = (0.4p)^2 - 2\times0.4p\times0.5q + (0.5q)^2$

(Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 0.16p^2 - 0.40pq + 0.25q^2$

(vi) Given: $(2xy + 5y)^2 = (2xy)^2 + 2\times 2xy\times 5y + (5y)^2$
(Since, $(a + b)^2 = a^2 + 2ab + b^2$)
$= 4x^2y^2 + 20xy^2 + 25y^2$

Q.4 Simplify

(i) $(a^2 - b^2)^2$
(ii) $(2x + 5)^2 - (2x - 5)^2$
(iii) $(7m - 8n)^2 + (7m - 8n)^2$
(iv) $(4m + 5n)^2 + (5m + 4n)^2$
(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
(vi) $(ab + bc)^2 - 2ab^2c$
(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

***Sol.* (i) Given:** $(a^2 - b^2)^2 = (a^2)^2 - 2\times a^2\times b^2 + (b^2)^2$ (Since, $(a-b)^2 = a^2 - 2ab + b^2$)
$= a^4 - 2a^2b^2 + b^4$

(ii) Given: $(2x + 5)^2 - (2x - 5)^2 = [(2x)^2 + 2\times 2x\times 5 + (5)^2] - [(2x)^2 - 2\times 2x\times 5 + (5)^2]$
(Since, $(a + b)^2 = a^2 + 2ab + b^2$ & $(a - b)^2 = a^2 - 2ab + b^2$)
$= [4x^2 + 20x + 25] - [4x^2 - 20x + 25]$
$= [4x^2 + 20x + 25 - 4x^2 + 20x - 25]$
$= 40x$

(iii) Given: $(7m - 8n)^2 + (7m + 8n)^2$
$= [(7m)^2 - 2\times 7m\times 8n + (8n)^2] + [(7m)^2 - 2\times 7m\times 8n + (8n)^2]$
(Since, $(a + b)^2 = a^2 + 2ab + b^2$ & $(a - b)^2 = a^2 - 2ab + b^2$)
$= [49m^2 - 112mn + 64n^2] + [49m^2 + 112mn + 64n^2]$
$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
$= 98m^2 + 128n^2$

(iv) Given: $(4m + 5n)^2 + (5m + 4n)^2$
$= [(4m)^2 + 2\times 4m\times 5n + (5n)^2] + [(5m)^2 + 2\times 5m\times 4n + (4n)^2]$
(Since, $(a + b)^2 = a^2 + 2ab + b^2$)
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80\,mn + 41n^2$

(v) Given: $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
$= [(2.5p)^2 - 2\times 2.5\text{p}\times 1.5q + (1.5q)^2] - [(1.5p)^2 - 2\times 1.5\text{p}\times 2.5q + (2.5q)^2]$
(Since, $(a - b)^2 = a^2 - 2ab + b^2$)
$= [6.25p^2 - 7.50pq + 2.25q^2] - [2.25p^2 - 7.50pq + 6.25q^2]$
$= 6.25p^2 - 7.50pq + 2.25q^2 - 2.25p^2 + 7.50pq - 6.25q^2$
$= 6.25p^2 - 2.25p^2 - 7.50pq + 7.50pq + 2.25q^2 - 6.25q^2$
$= 4p^2 - 4q^2$

(vi) Given: $(ab + bc)^2 - 2ab^2c = (ab)^2 + 2\times ab\times bc + (bc)^2 - 2ab^2c$
(Since, $(a + b)^2 = a^2 + 2ab + b^2$)
$= a^2b^2 + 2\times ab^2c + b^2c^2 - 2ab^2c$
$= a^2b^2 + b^2c^2$

(vii) Given: $(m^2 - n^2m)^2 + 2m^3n^2 = (m^2)^2 - 2\times m^2\times n^2m + (n^2m)^2 + 2m^3n^2$
(Since, $(a - b)^2 = a^2 - 2ab + b^2$)
$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$
$= m^4 + n^4m^2$

Q.5 Show that.

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$
(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$
(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$
(v) $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$

Sol. **(i) Given:** $(3x + 7)^2 - 84x = (3x - 7)^2$

By taking LHS = $3x + 7)^2 - 84x$

$= (3x)^2 + 2\times3x\times7 + (7)^2 - 84$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 + 42x + 49$

Now taking RHS = $(3x - 7)^2$

$= (3x)^2 - 2\times3x\times7 + (-7)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 9x^2 - 42x + 49$

From above, LHS = RHS

Hence Proved that $(3x + 7)^2 - 84x = (3x - 7)^2$

(ii) Given: $(9p-5q)^2 + 180pq = (9p + 5q)^2$

By taking LHS = $(9p - 5q)^2 + 180pq$

$= (9p)^2 - 2\times9p\times5q + (5q)^2 + 180pq$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

Now taking RHS = $(9p + 5q)^2$

$= (9p)^2 + 2\times9p\times5q + (5q)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 81p^2 + 90pq + 25q^2$

From above, LHS = RHS

Hence Proved that $(9p-5q)^2 + 180pq = (9p + 5q)^2$

(iii) Given: $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

By taking LHS = $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \left(\frac{4}{3}m\right)^2 - 2\left(\frac{4}{3}m\right)\left(\frac{3}{4}n\right) + \left(\frac{3}{4}n\right)^2 + 2mn$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2$

= RHS

Hence Proved that $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) Given: $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

By taking LHS = $(4pq + 3q)^2 - (4pq - 3q)^2$

$= [(4pq)^2 + 2\times4pq\times3q + (3q)^2] - [(4pq)^2 - 2\times4pq\times3q + (3q)^2]$

(Since, $(a + b)^2 = a^2 + 2ab + b^2$ & $(a - b)^2 = a^2 - 2ab + b^2$)

$= [16p^2q^2 + 24pq^2 + 9q^2] - [16p^2q^2 - 24pq^2 + 9q^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

$= 48pq^2$

= RHS

Hence Proved that $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) Given: $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

By taking LHS = $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$

Since, $(a - b)(a + b) = a^2 - b^2$

$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$

$= 0$

= RHS

Hence proved that $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Q.6 Using identities, evaluate.

(i) 71^2 **(ii) 99^2** **(iii) 102^2** **(iv) 998^2**

(v) 5.2^2 **(vi) 297×303** **(vii) 78×82** **(viii) 8.9^2**

(ix) 1.05×9.5

***Sol.* (i) Given:** $71^2 = (70 + 1)^2$

$= (70)^2 + 2 \times 70 \times 1 + (1)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 4900 + 140 + 1$

$= 5041$

(ii) Given: $99^2 = (100 - 1)^2$

$= (100)^2 - 2 \times 100 \times 1 + (1)^2$ (Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 10000 - 200 + 1$

$= 9801$

(iii) Given: $102^2 = (100 + 2)^2$

$= (100)^2 + 2 \times 100 \times 2 + (2)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 10000 + 400 + 4$

$= 10404$

(iv) Given: $998^2 = (1000 - 2)^2$

$= (1000)^2 - 2 \times 1000 \times 2 + (2)^2$(Since, $(a - b)^2 = a^2 - 2ab + b^2$)

$= 100000 - 4000 + 4$

$= 996004$

(v) Given: $5.2^2 = (5 + 0.2)^2$

$= (5)^2 + 2 \times 5 \times 0.2 + (0.2)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 25 + 2 + 0.04$

$= 27.04$

(vi) Given: $297\times303 = (300 - 3) \times (300 + 3)$

$= (300)^2 - (3)^2$ (Since, $(a - b)(a + b) = a^2 - b^2$)

$= 90000 - 9$

$= 89991$

(vii) Given: $78\times82 = (80 - 2) \times (80 + 2)$

$= (80)^2 - (2)^2$ (Since, $(a - b)(a + b) = a^2 - b^2$)

$= 6400 - 4$

$= 6396$

(viii) Given: $8.9^2 = (8 + 0.9)^2$

$= (8)^2 + 2 \times 8 \times 0.9 + (0.9)^2$ (Since, $(a + b)^2 = a^2 + 2ab + b^2$)

$= 64 + 14.4 + 0.81$

$= 79.21$

(ix) Given: $1.05 \times 9.5 = (1 + 0.05) \times (1 - 0.05) \times 10$

$= [(1)^2 - (0.05)^2] \times 10$ (Since, $(a - b)(a + b) = a^2 - b^2$)

$= 0.9975 \times 10$

$= 9.975$

Q.7 Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 - 49^2$ **(ii) $(1.02)^2 - (0.98)^2$**

(iii) $153^2 - 147^2$ **(iv) $12.1^2 - 7.9^2$**

***Sol.* (i) Given:** $51^2 - 49^2 = (51 + 49)(51 - 49)$ (Since, $a^2 - b^2 = (a + b)(a - b)$)

$= 100 \times 2$

$= 200$

(ii) Given: $(1.02)^2 - (0.98)^2 = (1.02 + 0.98)(1.02 - 0.98)$

(Since, $a^2 - b^2 = (a + b)(a - b)$)

$= 2.00 \times 0.04$

$= 0.08$

(iii) Given: $153^2 - 147^2 = (153 + 147)(153 - 147)$ (Since, $a^2 - b^2 = (a + b)(a - b)$)
$= 300 \times 6$
$= 1800$

(iv) Given: $12.1^2 - 7.9^2 = (12.1 + 7.9)(12.1 - 7.9)$ (Since, $a^2 - b^2 = (a + b)(a - b)$)
$= 20.0 \times 4.2$
$= 84$

Q.8 Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find
(i) 103×104 (ii) 5.1×5.2 (iii) 103×98 (iv) 9.7×9.8

***Sol.* (i) Given:** $103 \times 104 = (100 + 3) \times (100 + 4)$
$= (100)^2 + (3 + 4) \times 100 + 3 \times 4$
(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)
$= 10000 + 7 \times 100 + 12$
$= 10000 + 700 + 12$
$= 10712$

(ii) Given: $5.1 \times 5.2 = (5 + 0.1) \times (5 + 0.2)$
$= (5)^2 + (0.1 + 0.2) \times 5 + 0.1 \times 0.2$
(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)
$= 25 + 0.3 \times 5 + 0.02$
$= 25 + 1.5 + 0.02$
$= 26.52$

(iii) Given: $103 \times 98 = (100 + 3) \times (100 - 2)$
$= (100)^2 + (3 - 2) \times 100 + 3 \times (-2)$
(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)
$= 10000 + 100 - 6$
$= 10094$

(iv) Given: $9.7 \times 9.8 = (10 - 0.3) \times (10 - 0.2)$
$= (10)^2 + (-0.3) + (-0.2)) \times 10 + (-0.3) \times (-0.2)$
(Since, $(x + a)(x + b) = x^2 + (a + b)x + ab$)

$= 100 - 5 + 0.06$
$= 95.06$