Surface Area and Volume

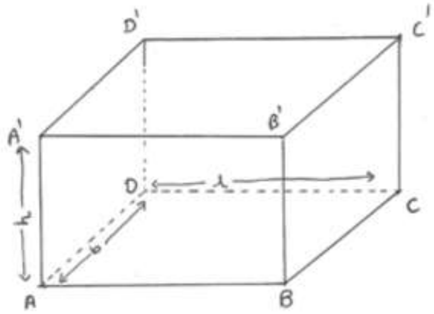
(1)*Cuboid:* If *l, b* and *h* denote respectively the length, breadth and height of a cuboid, then

(i) Total surface area of the cuboid = 2(lb + bh + lh) square units

(ii) Volume of the cuboid = Area of the base x height = lbh cubic units

(iii) Diagonal of the cuboid = $\sqrt{l^2+b^2+h^2}$ units.

(iv) Area of four walls of a room = $2(l + b)h$ sq. units.



For Example: Two cubes each of 10cm edge are joined end to end. Find the

(i) Surface area of the resulting cuboid

(ii) Volume of cuboid

(iii) Diagonal of the cuboid and

(iv) Area of four walls of a room

Solution: l = length of resulting cuboid = 10cm + 10cm = 20cm

b = breadth of resulting cuboid = 10cm

h = height of resulting cuboid = 10cm

(i) Total Surface Area of the cuboid = 2(lb + bh + lh) square units

$$A= 2\,(20 \times 10 + 10 \times 10 + 20 \times 10)\ cm^2$$
$$A= 1000\ cm^2.$$

(ii) Volume of the cuboid = lbh cubic units

$$V= 20 \times 10 \times 10\ cm^3$$
$$V= 2000\ cm^3.$$

(iii) Diagonal of the cuboid= $\sqrt{l^2+b^2+h^2}$

$$l=\sqrt{(20)^2+(10)^2+(10)^2}$$
$$l = 24.49\ cm.$$

(iv) Area of four walls of a room = 2 (l + b) h

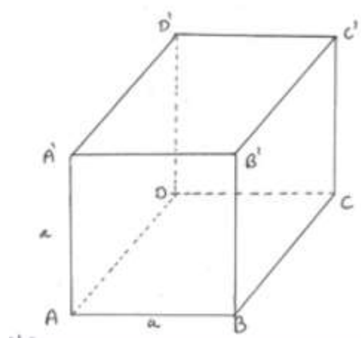
$$A = 2\,(20 + 10)\,10$$
$$A = 600\ cm^2$$

(2) *Cube:* If the length of each edge of a cube is *a* units, then

(i) Total surface area of the cube = $6a^2$ square units

(ii) Volume of the cube = a^3 cubic units.

(iii) Diagonal of the cube = $\sqrt{3}a$ units.



If the length of each edge of a cube is 4 cm units then find total surface area, volume of cube and diagonal of cube.

Solution: **(i)** Total Surface area= $6a^2$ cm^2

$A = 6(4)^2\ cm^2$

$A = 96\ cm^2$.

(ii) Volume of the cube = a^3 cm^3

$V = (4)^3\ cm^3$

$V = 64\ cm^3$.

(iii) Diagonal of the cube = $\sqrt{3}a$ cm

$l = \sqrt{3}(4)$ cm

$l = 6.92$ cm.

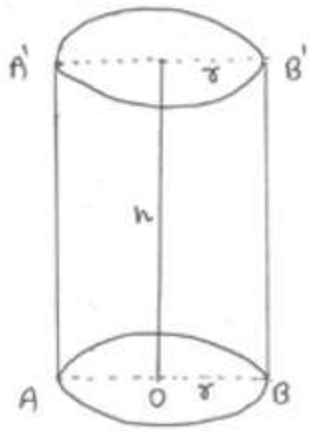
(3) *Right circular cylinder:* If r and h denote respectively the radius of the base and height of a right circular cylinder, then

(i) Area of each end = πr^2

(ii) Curved surface area of hollow cylinder = $2\pi rh$

(iii) Total surface area = $2\pi r\,(h + r)$

(iv) Volume = $\pi r^2 h$



For Example: The diameter of right circular cylinder is 6 cm and height is 9 cm. Find

(i) Area of each end

(ii) Curved surface area

(iii) Total surface area

(iv) Volume

Solution:

(i) Area of each end = π r^2

$A = \pi\,(3)^2$

$A = 3.14 \times (3)^2$

$A = 28.26\ cm^2$.

(ii) Curved surface area = $2\ \pi\ r\ h$

$A = 2 \times 3.14 \times 3 \times 9$

$A = 169.56\ cm^2$.

(iii) Total surface area = $2\ \pi\ r\ (h + r)$

$A = 2 \times 3.14 \times 3(9 + 3)$

$A = 226.08\ cm^2$.

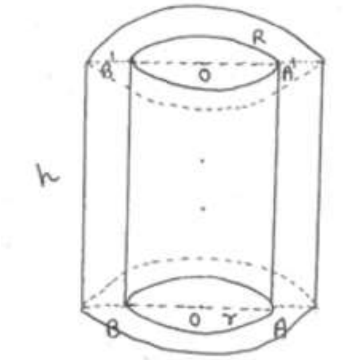
(iv) Volume = $\pi\ r^2\ h$

$V = 3.14 \times (3)^2 \times 9$

$V = 254.34\ cm^3$.

(4) *Right Circular Hollow Cylinder:* If R and r denote respectively the external and internal radii of a hollow right circular cylinder, then

(i) Area of each end= π ($R^2 - r^2$)
(ii) Curved surface area of hollow cylinder= $2\pi(R + r)h$
(iii) Total surface area= 2π $(R + r)(R + h - r)$
(iv) Volume of material= πh $(R^2 - r^2)$



For Example: The external and internal radii of a hollow cylinder is 8cm and 6cm respectively and height is 10 cm. Find
(i) Area of each end
(ii) Curved surface area
(iii) Total surface area
(iv) Volume

Solution:

(i) Area of each end = π $(R^2 - r^2)$

$$A = 3.14\ ((8)^2 - (6)^2)$$
$$A = 87.92\ cm^2.$$

(ii) Curved surface area = $2\pi h\ (R + r)$

$$A = 2 \times 3.14 \times 10 \times (8 + 6)$$
$$A = 879.2\ cm^2.$$

(iii) Total surface area = $2\pi\ (R + r)\ (R + h - r)$

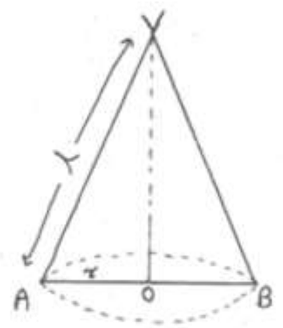
$$A = 2 \times 3.14 \times (8 + 6)\ (8 + 10 - 6)$$
$$A = 2 \times 3.14 \times 14 \text{ x } 12$$
$$A = 1055.04\ cm^2.$$

(iv) Volume of material = $\pi h\ (R^2 - r^2)$

$$V = 3.14 \times 10 \times ((8)^2 - (6)^2)$$
$$V = 879.2\ cm^3.$$

(5) *Right Circular Cone:* If r,h and l denote respectively the radius of base, height and slant height of a right circular cone, then

(i) $l^2 = r^2 + h^2$
(ii) Curved surface area = πrl
(iii) Total surface area = $\pi r^2 + \pi rl$
(iv) Volume = $\frac{1}{3}\pi r^2 h$



For Example: A right circular cone is of height 8.4 cm, radius of its base is 2.1 cm. Find
(i) Slant height
(ii) Curved surface area
(iii) Total surface area
(iv) Volume
Solution: (i) Slant height $l^2 = r^2 + h^2$

$$l^2 = (2.1)^2 + (8.4)^2$$
$$l^2 = 74.97 \text{ cm}^2$$
$$l = 8.66 \text{ cm.}$$

(ii) Curved surface area = $\pi r l$

$$A = 3.14 \times 2.1 \times 8.66$$
$$A = 57.10 \text{ cm}^2.$$

(iii) Total surface area = $\pi r(l + r)$

$$A = 3.14 \times 2.1\,(8.66 + 2.1)$$
$$A = 70.95 \text{ cm}^2.$$

(iv) Volume = $1/3\ \pi r^2 h$

$$V = 1/3 \times 3.14 \times (2.1)^2 \times 8.4$$
$$V = 38.77 \text{ cm}^3.$$

(6) *Sphere:* For a sphere of radius r, we have
(i) Surface area = $4\pi r^2$
(ii) Volume = $\frac{4}{3}\pi r^3$.

For Example: The radius of sphere is 4 cm then find the surface area and volume.
Solution: **(i)** Surface area = $4\pi r^2$

$$A = 4 \times 3.14 \times (4)^2$$
$$A = 200.96 \text{ cm}^2.$$

(ii) Volume = $4/3\ \pi r^3$

$$V = 4/3 \times 3.14 \times (4)^3$$
$$V = 66.99 \text{ cm}^3.$$

(7) *Frustum of a right circular cone:* If h is the height, l the slant height and r1 and r2 the radii of the circular bases of a frustum of a cone, then

(i) Volume of the frustum = $\frac{\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)h$

(ii) Lateral surface area = $\pi(r_1 + r_2)l$

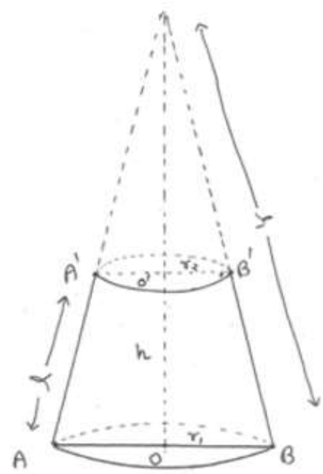
(iii) Total surface area = $\pi\{(r_1 + r_2)l + r_1^2 + r_2^2\}$

(iv) Slant height of the frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$

(v) Height of the cone of which the frustum is a part = $\frac{hr_1}{r_1 - r_2}$

(vi) Slant height of the cone of which the frustum is a part = $\dfrac{lr_1}{r_1 - r_2}$

(vii) Volume of the frustum = $\dfrac{h}{3}\{A_1 + A_2 + \sqrt{A_1A_2}\}$, where Al and Az denote the areas of circular bases of the frustum.



For Example: If 4 cm is the height, 4.12 cm is the slant height and 2.5 cm and 1.5 c, is radii of the circular bases of a frustum of cone, then find
(i) Volume of the frustum
(ii) Lateral surface area
(iii) Total surface area
(iv) Slant height of frustum
(v) Height of the cone of which the frustum is a part
(vi) Slant height of the cone of which the frustum is a part
(vii) Volume of the frustum

Solution: **(i)** Volume of the frustum = $\pi/3\ (r_1^2 + r_1r_2 + r_2^2)\ h$

$$V = \pi/3\ ((2.5)^2 + 2.5 \times 1.5 + (1.5)^2)\ 4$$
$$V = \pi/3 \times 12.25 \times 4$$
$$V = 51.28\ cm^3.$$

(ii) Lateral surface area = $\pi\ (r_1 + r_2)\ l$

$$A = 22/7 \times (2.5 + 1.5) \times 4.13$$
$$A = 51.74\ cm^2.$$

(iii) Total surface area = $\pi\ ((r_1 + r_2)\ l + r_1^2 + r_2^2)$

$$A = 3.14 \times ((2.5 + 1.5)\ 4.13 + (2.5)^2 + (1.5)^2)$$
$$A = 78.56\ cm^2.$$

(vi) Slant height of the cone of which the frustum is a part $l = l.r_1 / (r_1.\ r_2)$

$$l = (4.13 \times 2.5)/(2.5 \times 1.5)$$
$$l = 10.325\ cm.$$

(v) Height of the cone of which the frustum is a part $H = h.r_1/(r_1 - r_2)$

$$H = 4 \times 2.5/(2.5 - 1.5)$$
$$H = 10/1$$
$$H = 10\ cm.$$

(vi) Slant height of the cone of which the frustum is a part $l = l.r_1 / (r_1 - r_2)$

$$l = 4.13 \times 2.5 / (2.5 - 1.5)$$
$$l = 10.325\ cm.$$

(vii) Volume of the frustum = $h/3\ (A_1 + A_2 + \sqrt{(A_1A_2)})$

$$A_1 = \pi\ r_1^2 = 3.14 \times (2.5)^2 = 19.625$$
$$A_2 = \pi\ r_2^2 = 3.14 \times (1.5)^2 = 7.065$$
$$V = 4/3\ (19.625 + 7.065 + \sqrt{(19.625 \times 7.065)})$$
$$V = 4/3\ (26.69 + \sqrt{138.65})$$
$$V = 51.28\ cm^3.$$