Areas Related to Circles

(1) For a circle of a radius r, we have

(i) Circumference = 2πr
(ii) Area = πr2
(iii) Area of semi-circle = $\frac{\pi r^2}{2}$
(iv) Area of a quadrant = $\frac{\pi r^2}{4}$

For Example: Find circumference and area of a circle of radius 4.2 cm
Solution: We know that the circumference C and area A of a Circle of radius r given by, C = 2πr and A = πr2 respectively.

(i) Circumference of the circle C = 2πr

$$= 2 \times \frac{22}{7} \times (4.2) = 26.4\text{cm}$$

(ii) Area of the circle A=πr2

$$= \frac{22}{7} \times (4.2)^2 \Rightarrow 55.4\text{cm}^2$$

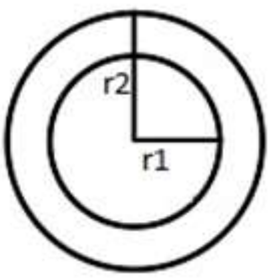
Hence, Circumference of the circle and area of the circle and area of the circle are 26.4cm and 55.4cm² respectively.

(iii) Area of a semi-circle = $\frac{\pi r^2}{2} = \frac{22 \times (4.2)^2}{7 \times 2} = 27.2\ cm^2$

(iv) Area of a quadrant = $\frac{\pi r^2}{4} = \frac{22 \times (4.2)^2}{7 \times 4} = 13.86\ cm^2$

(2) If R and r are the radii of two concentric circles such that R > r then, Area enclosed by the two circles = $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

For Example: The area enclosed between the concentric circle is 770 cm². If the radius of the outer circle is 21 cm, find the radius of the inner circle.



Solution: Let the radius of inner and outer radius be r1 and r2 respectively.
It is given that area that area enclosed between concentric circles is 770 cm2
Radius of the outer circle is 21 cm
Then, area enclosed between the concentric circle =πr22−πr21

$\Rightarrow \pi r_2^2 - \pi r_1^2 = 770$

$\Rightarrow \pi\ ((21) - r_1^2) = 770$

$\Rightarrow (441 - r_1^2) = \frac{770 \times 7}{22}$

$\Rightarrow r_1^2 = 441 - 245 = 196$

$\Rightarrow r_1 = 14$

Hence, the radius of the inner circle is 14 cm.

(3) If a sector of a circle of radius r contains an angle of θ Then,

(i) Length of the arc of the sector = $\frac{\theta}{360} \times 2\pi r = \frac{\theta}{360} \times (\text{Circumference})$

For Example: Find the Length of the arc of the sector that subtends an angle of 30∘ at the centre of a circle of radius 4 cm.

Solution: The length of the arc is given by $l = \frac{\theta}{360} \times 2\pi r$

Here, r = 4 cm and θ = 30∘

$\Rightarrow l = (\frac{\theta}{360} \times 2\pi r \times 4)$

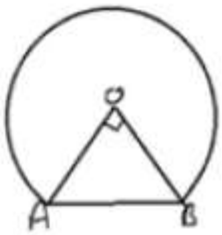
$\Rightarrow l = \frac{2\pi}{3} cm$

Hence, the length of the arc is 2π3 cm

(ii) Perimeter of the sector= $2r + \frac{\theta}{360} \times 2\pi r$

For Example: The cross section of railway tunnel the radius of the circular part is 2m. If ∠AOB = 90∘ calculate the perimeter of the cross section.

Solution:



We have OA = 2m

Now using Pythagoras theorem in ΔAOB, AB = $\sqrt{2^2 + 2^2} = 2\sqrt{2}m$

Let the height of the tunnel be h

Area of ΔAOB = $\frac{1}{2} \times 2 \times 2 = \frac{1}{2} \times 2\sqrt{2} \times OM = 2$

OM = √2

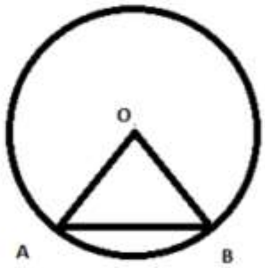
h = (2 + √2)cm

Perimeter of cross-section is = major arc AB + AB = (2π × 2 × $\frac{3}{4}$) + 2√2 = (3π + 2√2)cm

(iii) Area of the sector= $\frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360}$ **× (Area of the circle)**

For Example: AB is a chord of a circle with centre O and radius 4 cm. AB is of length 4 cm and divided the circle into two segments find the area of the minor segment

Solution:



It is given that chord AB divides the circle into two segments
In ΔAOB
OA = OB = 4cm

AM = $\frac{AB}{2}$ = 2cm

Let ∠AOB = 2θ, then, ∠AOM = ∠BOM = θ
In ∠OAM we have

$$\sin\theta = \frac{AM}{AO} = \frac{2}{4} = \frac{1}{2}$$

$$\theta = \sin^{-1}\frac{1}{2}$$

= 30°
Hence, ∠AOB=2θ=2×30°=60°
We know that the area of minor segment of angle θ in a circle of radius r is

$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$$

Now, using the value of r and θ we can find the area of minor segment

$$A = \left\{\frac{\pi\theta}{360} - \sin\frac{60^\circ}{2}\cos\frac{60^\circ}{2}\right\}(4)^2$$

$$\Rightarrow A = \left\{\frac{\pi}{6} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right\}(4)^2$$

$$A = \left\{\frac{8\pi}{3} - 4\sqrt{3}\right\}cm^2$$

Hence, area of minor segment is $\left\{\frac{8\pi}{3} - 4\sqrt{3}\right\}cm^2$

(iv) Area of the segment = Area of the corresponding sector – Area of the corresponding triangle

$$= \frac{\theta}{360}\times\pi r^2 - r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \left\{\frac{\pi\theta}{360} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$$

For Example: The radius of a circle with centre O is 5 cm. two radii OA and OB are drawn at right angles to each other. Find the areas of segment made by chord AB.

Solution: Radius of the circle = 5 cm

Area of the minor segment AB = $\left(\frac{\pi\theta}{360^\circ} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)r^2$

$$AB = \left(\frac{3.14\times 90^\circ}{360^\circ} - \sin 45^\circ\cos 45^\circ\right)(5)^2$$

$$= \left(\frac{282.6}{360^\circ} - \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right)(5)^2 \quad = 7.125\ cm^2$$

Area of minor segment = area of circle – area of minor segment = $\pi r^2 - 7.125$
=3.14−25−7.125 = 71.37 cm2