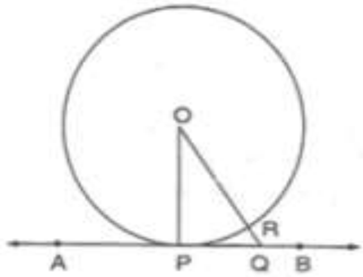
Circles

(1) Prove that Tangent to a circle at a point is perpendicular to the radius through the point of contact.

Given: A circle C(O, r) and a tangent AB at a point P.

To Prove: OP⊥ AB

Construction: Take any Point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.



Proof: We know that among all line segments joining the point O to a point on AB the shortest one is perpendicular to AB. So, to prove that OP⊥ AB, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Cleraly, OP = OR [Radii of the same circle]

Now, OQ = OR + RQ

⇒ OQ > OR

⇒ OQ > OP [OP = OR]

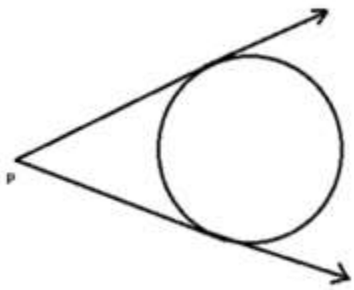
⇒ OQ < OQ

Thus, OP is shorter than any other segment joining O to any point of AB.

Hence, OP⊥ AB.

(2) Prove that from a point, lying outside a circle, two and only two tangents can be drawn to it.

When the point lies outsides the circle, there are exactly two tangents to circle from a point which lies outside the circle. As shown in figure.

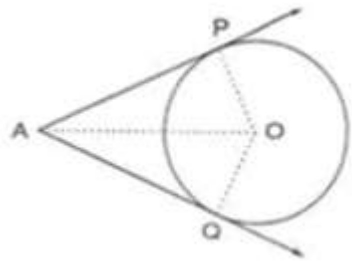


(3) Prove that the lengths of the two tangents drawn from an external point to a circle are equal.

Given: AP and AQ are two tangents from a point A to a circle C(O, r).

To Prove: AP = AQ

Construction: Join OP, OQ and OA



Proof: In order to prove that AP = AQ we shall first prove that ΔOPA ≅ ΔOQA.
Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.
OP⊥ AB and OQ⊥ AQ
⇒ ∠OPA = ∠OQA = 90°.... (i)
Now, in right triangle OPA and OQA, we have
OP = OQ [Radii of a circle]
∠OPA = ∠OQA [From (i)]
And, OA = OA [Common]
So, by RHS-Criterion of congruence, we get
ΔOPA ≅ ΔOQA
⇒ AP = AQ