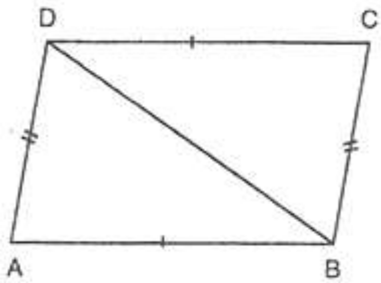
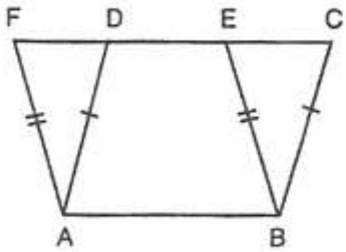
Area of Parallelogram

(1) Prove that a diagonal of a parallelogram divides it into two triangles of equal area.
Given: A parallelogram ABCD in which BD is one of the diagonals.
To prove: ar(ΔABD) = ar(ΔCDB)



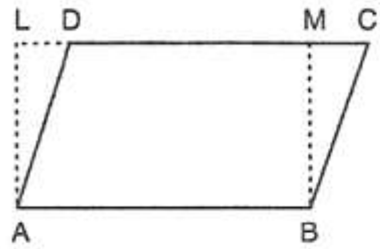
Proof: Since two congruent geometrical figures have equal area. Therefore, in order to prove that ar(ΔABD) = ar(ΔCDB) it is sufficient to show that
ΔABD ≅ ΔCDB
In Δs ABD and CDB, we have
AB = CD
AD = CB
And, BD = DB
So, by SSS criterion of congruence, we have
ΔABD ≅ ΔCDB
Hence, ar(ΔABD) = ar(ΔCDB)

(2) Prove that parallelograms on the same base and between the same parallels are equal in area.
Given: Two parallelograms ABCD and ABEF, which have the same base AB and which are between the same parallel lines AB and FC.
To prove: ar(parallelogramABCD) = ar(parallelogramABCD)



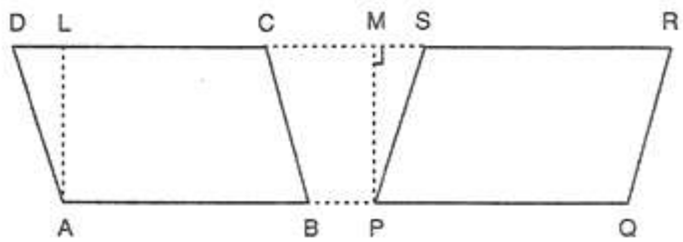
Proof: In Δs ADF and BCE, we have
AD = BC
AF = BE
And, ∠DAF = ∠CBE [∵ AD ∥ BC and AF ∥ BE]
So, by SAS criterion of congruence, we have
ΔADF ≅ ΔBCE
ar(ΔADF) = ar(ΔBCE) .....(i)
Now, ar(parallelogram ABCD) = ar(sq. ABED) + ar(ΔBCE)
ar(parallelogram ABCD) = ar(sq. ABED) + ar(ΔADF) [Using(i)]
ar(parallelogram ABCD) = ar(parallelogram ABEF)
Hence, ar(parallelogram ABCD) = ar(parallelogram ABEF)

(3) Prove that the area of a parallelogram is the product of its base and the corresponding altitude.
Given: A parallelogram ABCD in which AB is the base and AL the corresponding altitude.
To prove: ar(parallelogram ABCD) = AB × AL
Construction: Complete the rectangle ALMB by drawing BM⊥CD.



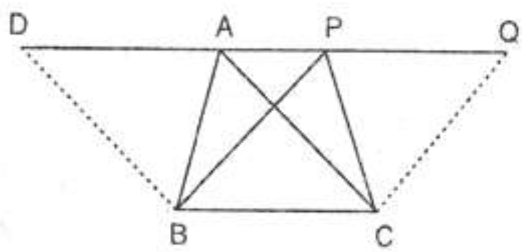
Proof: Since ar(parallelogram ABCD) and rectangle ALMB are on the same base and between the same parallels.
ar(parallelogram ABCD)
= ar(rect. ALMB)
= AB×AL [By rect. Area axiom area of a rectangle = Base X Height]
Hence, ar(parallelogram ABCD) = AB × AL

(4) Prove that parallelograms on equal bases and between the same parallels are equal in area.
Given: Two parallelograms ABCD and PQRS with equal bases AB and PQ and between the same parallels AQ and DR.
To prove: ar(parallelogram ABCD) = ar(parallelogram PQRS)
Construction: Draw AL⊥DR and PM⊥DR



Proof: Since AB ⊥ DR, AL ⊥ DR and PM ⊥ DR
AL = PM
Now, ar(parallelogram ABCD) = AB × AL
ar(parallelogram ABCD) = PQ × PM [AB = PQ and AL = PM]
ar(parallelogram ABCD) = ar(parallelogram PQRS)

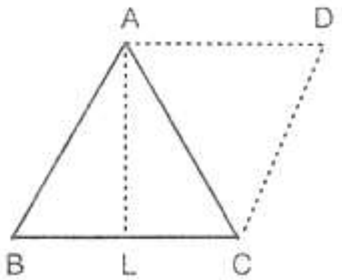
(5) Prove that triangles on the same bases and between the same parallels are equal in area.
Proof: We have,
BD ∥ CA
And, BC ∥ DA
sq. BCAD is a parallelogram.



Similarly, sq. BCQP is a parallelogram.

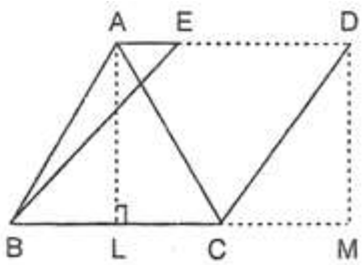
Now, parallelograms ECQP and BCAD are on the same base BC, and between the same parallels.
ar(parallelogram BCQP) = ar(parallelogram BCAD) ....(i)
We know that the diagonals of a parallelogram divides it into two triangles of equal area.
ar(ΔPBC) = 12ar(parallelogram BCQP) .....(ii)
And, ar(ΔABC) = 12ar(parallelogram BCAQ) ....(iii)
Now, ar(parallelogram BCQP) = ar(parallelogram BCAD) [From (i)]
12ar(parallelogram BCQP) = 12ar(parallelogram BCAD)
ar(ΔABC) = ar(ΔPBC) [From (ii) and (iii)]
Hence, ar(ΔABC) = ar(ΔPBC)

(6) Prove that the area of a triangle is half the product of any of its sides and the corresponding altitude.
Given: A ΔABC in which AL is the altitude to the side BC.
To prove: ar(ΔABC) = 12(BC × AL)
Construction: Through C and A draw CD ∥ BA and AD ∥ BC respectively, intersecting each other at D.



Proof: We have,
BA ∥ CD
And, AD ∥ BC
BCDA is a parallelogram.
Since AC is a diagonal of parallelogram BCDA.
ar(ΔABC) = 12ar(parallelogram BCAD)
ar(ΔABC) = 12(BC×AL) [BC is the base and AL is the corresponding altitude of parallelogram BCDA]

(7) Prove that if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to the half of the parallelogram.
Given: A ΔABC and a parallelogram BCDE on the same base BC and between the same parallel BC and AD.
To prove: ar(ΔABC) = 12ar(parallelogram BCDE)
Construction: Draw AL⊥BC and DM⊥BC, meeting BC produced in M.



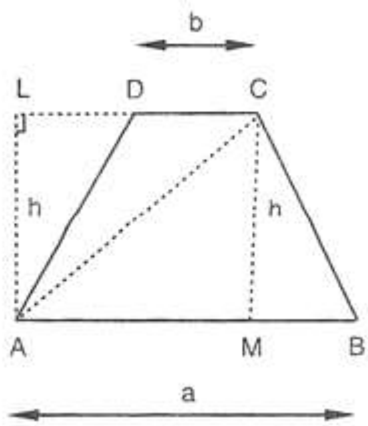
Proof: Since A, E and D are collinear and BC ∥ AD
AL = DM .....(i)
Now,
ar(ΔABC) = 12(BC × AL)
ar(ΔABC) = 12(BC × DM) [AL = DM (from (i)]
ar(ΔABC) = 12ar(parallelogram BCDE)

(8) Prove that the area of a trapezium is half the product of its height and the sum of parallel sides.

Given: A trapezium ABCD in which AB ∥ DC; AB = a, DC = b and AL = CM = h, where AL ⊥ DC and CM ⊥ AB

To prove: ar(trap. ABCD) = 12h × (a+b)

Construction: Join AC



Proof: We have,

ar(trap. ABCD) = ar(ΔABC) + ar(ΔACD)

ar(trap. ABCD) = 12(AB × CM) + 12(DC × AL)

ar(trap. ABCD) = 12ah × 12bh [AB = a and DC = b]

ar(trap. ABCD) = 12h × (a+b)

(9) Prove that triangles having equal areas and having one side of one of the triangles, equal to one side of the other, have their corresponding altitudes equal.

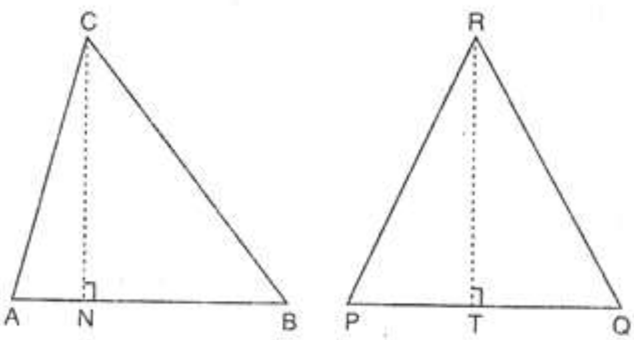
Given: Two triangles ABC and PQR such that:

ar(ΔABC) = ar(ΔPQR)

AB = PQ

CN and RT are the altitudes corresponding to AB and PQ respectively of the two triangles.

To prove: CN = RT



Proof: In ΔABC, CN is the altitude corresponding to side AB.

ar(ΔABC) = 12(AB × CN) ....(i)

Similarly, we have,

ar(ΔPQR) = 12(PQ × RT) .....(ii)

Now, ar(ΔABC) = ar(ΔPQR)

12(AB × CN) = 12(PQ × RT)

(AB × CN) = (PQ × RT)

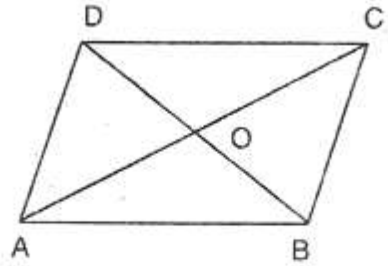
(PQ × CN) = (PQ × RT) [AB = PQ (Given)]

CN = RT

(10) Prove that if each diagonal of a quadrilateral separates it into two triangles of equal area, then the quadrilateral is a parallelogram.

Given: A quadrilateral ABCD such that its diagonals AC and BD are such that ar(ΔABD) = ar(ΔCDB) and ar(ΔABC) = ar(ΔACD)

To prove: Quadrilateral ABCD is a parallelogram.

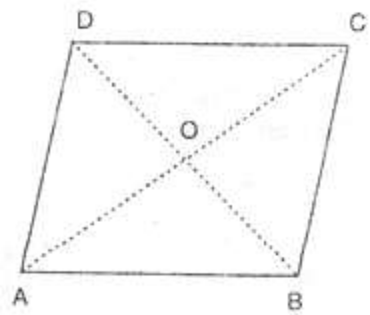


Proof: Since diagonal AC of the quadrilateral ABCD separates it into two triangles of equal area. Therefore,
ar(ΔABC) = ar(ΔACD) .....(i)
But, ar(ΔABC) + ar(ΔACD) = ar(quad. ABCD)
2ar(ΔABC) = ar(quad. ABCD) [Using (i)]
ar(ΔABC) = 12ar(quad. ABCD) ....(ii)
Since diagonal BD of the quadrilateral ABCD separates it into triangles of equal area.
ar(ΔABD) = ar(ΔBCD) ....(iii)
But, ar(ΔABD) + ar(ΔBCD) = ar(quad. ABCD)
2ar(ΔABD) = ar(quad. ABCD) [Using (iii)]
ar(ΔABD) = 12ar(quad. ABCD) .....(iv)
From (ii) and (iv), we get
ar(ΔABC) = ar(ΔABD)
Since Δs ABC and ABD are on the same base AB. Therefore they must have equal corresponding altitudes.
i.e. Altitude from C of ΔABC = Altitude from D of ΔABD
DC ∥ AB
Similarly, AD ∥ BC
Hence, quadrilateral ABCD is a parallelogram.

(11) Prove that the area of a rhombus is half the product of the lengths of its diagonals.

Given: A rhombus ABCD whose diagonals AC and BD intersect at O.

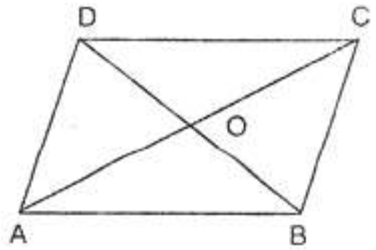
To prove: ar(rhombus ABCD) = 12(AC × BD)



Proof: Since the diagonals of a rhombus intersect at right angles. Therefore,
OB ⊥ AC and OD ⊥ AC
ar(rhombus) = ar(ΔABC) + ar(ΔADC)
ar(rhombus) = 12(AC × BO) + 12(AC × DO)

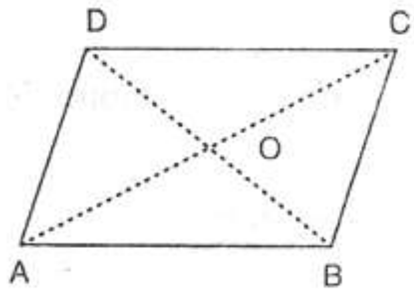
ar(rhombus) = 12(AC × (BO + DO))
ar(rhombus) = 12(AC × BD)

(12) Prove that diagonals of a parallelogram divide it into four triangles of equal area.
Given: A parallelogram ABCD. The diagonals AC and BD intersect at O.
To prove: ar(ΔOAB) = ar(ΔOBC) = ar(ΔOCD) = ar(ΔAOD)



Proof: Since the diagonals of a parallelogram bisect each other at the point of intersection.
OA = OC and OB = OD
Also, the median of a triangle divides it into two equal parts.
Now, in ΔABC, BO is the median.
ar(ΔOAB) = ar(ΔOBC) ....(i)
In ΔBCD, CO is the median
ar(ΔOBC) = ar(ΔOCD) .....(ii)
In ΔACD, DO is the median
ar(ΔOCD) = ar(ΔAOD) ....(iii)
From (i), (ii) and (iii), we get
ar(ΔOAB) = ar(ΔOBC) = ar(ΔOCD) = ar(ΔAOD)

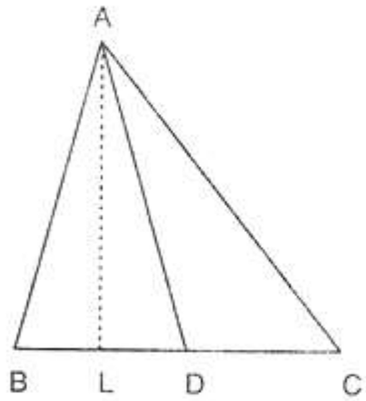
(13) Prove that if the diagonals AC and BD of a quadrilateral ABCD, intersect at O and separate the quadrilateral into four triangles of equal area, then the quadrilateral ABCD is parallelogram.
Given: A quadrilateral ABCD such that its diagonals AC and BD intersect at O and separate it into four parts such that
ar(ΔOAB) = ar(ΔOBC) = ar(ΔOCD) = ar(ΔAOD)
To prove: Quadrilateral ABCD is a parallelogram.



Proof: We have,
ar(ΔAOD) = ar(ΔBOC)
ar(ΔAOD) + ar(ΔAOB) = ar(ΔBOC) + ar(ΔAOB)
ar(ΔABD) = ar(ΔABC)
Thus, Δs ABD and ABC have the same base AB and have equal areas. So, their corresponding altitudes must be equal.
Altitude from ΔABD Altitude from C of ΔABC
DC ∥ AB

similarly, we have, AD ∥ BC.
Hence, quadrilateral ABCD is a parallelogram.

(14) Prove that a median of a triangle divides it into two triangles of equal area.
Given: A ΔABC in which AD is the median.
To prove: ar(ΔABD) = ar(ΔADC)
Construction: Draw AL ⊥ BC.



Proof: Since AD is the median of ΔABC. Therefore, D is the midpoint of BC.
BD = DC
BD × AL = DC × AL [Multiplying both sides by AL]
12(BD × AL) = 12(DC × AL)
ar(ΔABD) = ar(ΔADC)
ALITER Since Δs ABD and ADC have equal bases and the same altitude AL.
ar(ΔABD) = ar(ΔADC)