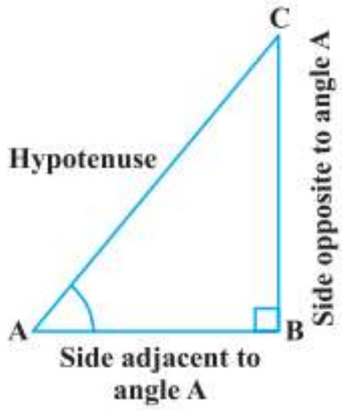
Introduction to Trigonometry

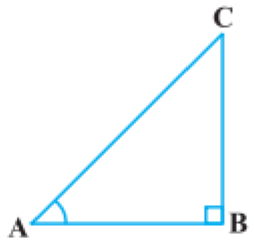
(1) For a right angled triangle ABC, side BC is called the side opposite to angle A, AC is called the hypotenuse and AB is called the side adjacent to angle A.



(2) The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and the length of its sides. The trigonometric ratios of angle A in right triangle ABC are defined as follows:

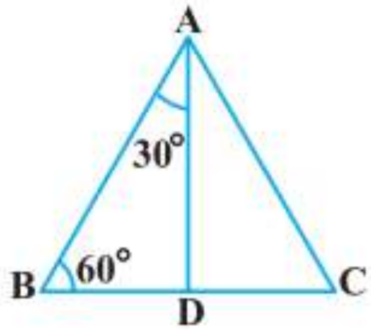
(i) sine of ∠ A = (side opposite to angle A)/ (hypotenuse) = BC/AC
(ii) cosine of ∠ A = (side adjacent to angle A)/ (hypotenuse) = AB/AC
(iii) tangent of ∠ A = (side opposite to angle A)/ (side adjacent to angle A) = BC/AB
(iv) cosecant of ∠ A = 1/(sine of ∠ A) = (hypotenuse)/ (side opposite to angle A) = AC/BC
(v) secant of ∠ A = 1/(cosine of ∠ A) = (hypotenuse)/ (side adjacent to angle A) = AC/AB
(vi) cotangent of ∠ A = 1/(tangent of ∠ A) = (side adjacent to angle A)/ (side opposite to angle A) = AB/BC

(3) Trigonometric Ratios of 45°:



In Δ ABC, right-angled at B, if one angle is 45°, then the other angle is also 45°, i.e., ∠A = ∠C = 45°.
Now, let BC = AB = a.
Applying the Pythagoras Theorem, we get,
$AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$,
So, AC = $a\sqrt{2}$
As per the trigonometric ratio definitions, we have,
(i) sin 45° = (side opposite to angle 45°)/(hypotenuse) =BC/AC = $a/ a\sqrt{2} = 1/\sqrt{2}$
(ii) cos 45° = (side adjacent to angle 45°)/(hypotenuse) = AB/AC = $a/ a\sqrt{2} = 1/\sqrt{2}$
(iii) tan 45° = (side opposite to angle 45°)/(side adjacent to angle 45°) =BC/AB = a/ a = 1
(iv) cosec 45° = 1/(sin 45°) = (hypotenuse)/ (side opposite to angle 45°) = AC/BC =$\sqrt{2}$
(v) sec 45° = 1/(cos 45°) = (hypotenuse)/ (side adjacent to angle 45°) = AC/AB =$\sqrt{2}$
(vi) cot 45° = 1/(tan 45°) = (side adjacent to angle 45°)/ (side opposite to angle 45°) = AB/BC = 1.

(4) Trigonometric Ratios of 30° and 60°:



Consider an equilateral triangle ABC. Since each angle in an equilateral triangle is 60°, therefore, ∠A = ∠B = ∠C = 60°. And draw a perpendicular AD from A to side BC.
Here, Δ ABD ≅ Δ ACD. Therefore, BD = DC and ∠ BAD = ∠ CAD (as per CPCT)
From the figure, it can be seen that, ∠ ABD = 60° and ∠ BAD = ½ ∠ BAC = ½ x 60° = 30°
Now, let us assume AB= BC = CA = 2a. Therefore, BD = ½ BC = a.
Applying Pythagoras theorem in Δ ABD, we get,
$AD^2 = AB^2 - BD^2 = (2a)^2 - (a)^2 = 3a^2$
Hence, AD = a√3
As per the trigonometric ratio definitions, we have,
(i) sin 30° = BD/AB = a/2a = ½
(ii) cos 30° = AD/AB = √3a /2a = √3/2
(iii) tan 30° = BD/AD = a/a√3 = 1/√3
(iv) cosec 30° = 1/ sin 30° = 2
(v) sec 30° = 1/ cos 30° = 2/√3
(vi) cot 30° = 1/ tan 30° =√3

Similarly, as per the trigonometric ratio definitions, we have,
(i) sin 60° =√3/2
(ii) cos 60° = 1/2
(iii) tan 60° =√3
(iv) cosec 60° = 2√3
(v) sec 60° = 2
(vi) cot 60° = 1/√3

(5) Trigonometric Ratios of 0°:
(i) sin 0° = 0
(ii) cos 0° = 1
(iii) tan 0° = 0
(iv) cosec 0° = Not defined
(v) sec 0° = 1
(vi) cot 0° = Not defined

(6) Trigonometric Ratios of 90°:
(i) sin 90° = 1
(ii) cos 90° = 0
(iii) tan 90° = Not defined
(iv) cosec 90° = 1
(v) sec 90° = Not defined
(vi) cot 90° = 0

(7) Table representing the trigonometric ratios of 0°, 30°, 45°, 60° and 90°:

∠ A	0°	30°	45°	60°	90°
sin A	0	1/2	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	1/2	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot a	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

***For Example*:** Evaluate $2 \tan^2 45° + \cos^2 30° – \sin^2 60°$
Now, $2 \tan^2 45° + \cos^2 30° – \sin^2 60°$
$= 2 (1)^2 + (\sqrt{3}/2)^2 – (\sqrt{3}/2)^2$
$= 2 + 3/4 – 3/4$
$= 2$.

(8) Trigonometric Ratios of Complementary Angles:

(i) $\sin (90° – A) = \cos A$
(ii) $\cos (90° – A) = \sin A$
(iii) $\tan (90° – A) = \cot A$
(iv) $\cot (90° – A) = \tan A$
(v) $\sec (90° – A) = \operatorname{cosec} A$
(vi) $\operatorname{cosec} (90° – A) = \sec A$

***For Example*:** Simplify tan 26° / cot 64°.
(i) We know that, cot A = tan (90° – 64°) = cot 64°.
(ii) Therefore, tan 26° / cot 64° = cot 64°/ cot 64° = 1.

(9) Trigonometric Identities:

(i) $\cos^2 A + \sin^2 A = 1$
(ii) $1 + \tan^2 A = \sec^2 A$
(iii) $\cot^2 A + 1 = \operatorname{cosec}^2 A$

***For Example*:** Evaluate sin 25° cos 65° + cos 25° sin 65°.
=sin 25° cos 65° + cos 25° sin 65°
= sin 25° cos (90° – 25°) + cos 25° sin(90° – 25°)
$= \sin^2 25° + \cos^2 25°$
$= 1$ (Since $\cos^2 A + \sin^2 A = 1$)

***For Example*:** Prove that (cosec A – sin A)(sec A – cos A) = 1/(tan A + cot A)
LHS = (cosec A – sin A)(sec A – cos A)

$= (1/\sin A - \sin A)(1/\cos A - \cos A)$
$= ((1 - \sin^2 A)/\sin A)\ ((1 - \cos^2 A)/\cos A)$
$= (\cos^2 A \times \sin^2 A)/(\sin A \cos A)$
$= \sin A \cos A$
RHS $= 1/(\tan A + \cot A)$
$= 1/(\sin A/\cos A + \cos A/\sin A)$
$= 1/((\sin^2 A + \cos^2 A)/(\sin A \cos A))$
$= (\sin A \cos A)/(\sin^2 A + \cos^2 A)$
$= \sin A \cos A$
Thus, LHS = RHS.