Coordinate Geometry

(1) The abscissa and ordinate of a given point are the distances of the point from y-axis and x-axis respectively.

(2) The coordinates of any point on x-axis are of the form (x, 0).

(3) The coordinates of any point on y-axis are of the form (0, y).

(4) The distance between points P(x1, y1) and Q(x2, y2) is given by,

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For Example:

If P(−6, 7) and Q(−1, −5), then distance between these two point is given as follow:

Here, $x_1 = -6$, $y_1 = 7$

$x_2 = -1$, $y_2 = -5$

Let O be the distance between two points (−6, 7) and (−1, −5).

The distance between two points is given by

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-1-(-6))^2 + (-5-7)^2}$

= $\sqrt{(-1+6)^2 + (-12)^2}$

= $\sqrt{(5)^2 + (-12)^2}$

= $\sqrt{25+144}$

= $\sqrt{169}$

= 13 units

Hence the distance between two points is 13 units.

(5) Distance of a point P(x, y) from the origin O(0, 0) is given by OP = $\sqrt{x^2 + y^2}$

For Example: P(−6,7) and O(0,0) is given then distance between them i.e. OP is calculated as follow:

OP = $\sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$

OP = $\sqrt{x^2 + y^2}$

OP = $\sqrt{6^2 + 7^2}$

OP = $\sqrt{85}$

(6) The coordinates of the point which divides the join of points P(x_1, y_1) and Q(x_2, y_2) internally in the ration m:n are $\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$

For Example: Find the coordinates of the point which is divided the line segment joining (-1, 3) and (4,-7) internally in the ratio 3:4.
Solution: Let the end points of AB be A(-1, 3) and B(4, -7).
($x_1 = -1$, $y_1 = 3$) and ($x_2 = 4$, $y_2 = -7$)
Also, m = 3 and n = 4

Let P(x, y) be the required point, then by section formula, we have $x = \dfrac{mx_2 + nx_1}{m+n}$, $y = \dfrac{my_2 + ny_1}{m+n}$

$\Rightarrow \quad x = \dfrac{3\times 4 + 4\times(-1)}{3+4}, y = \dfrac{3\times(-7)+4\times 3}{3+4}$

$\Rightarrow \quad x = \dfrac{12-4}{7}, y = \dfrac{-21+12}{7}$

$\Rightarrow \quad x = \dfrac{8}{7}, y = \dfrac{-9}{7}$

Hence, the required point is P $\left(\dfrac{8}{7}, \dfrac{-9}{7}\right)$

(7) The coordinates of the mid-point of the line segment joining the points P($x1$, $y1$) and Q($x2$, $y2$) are $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

For Example: P(4, 8) and Q(6, 10)

Mid – Point = $\left(\dfrac{4+6}{2}, \dfrac{8+10}{2}\right)$

So mid-point of PQ is M(5, 9).

(8) The coordinates of the centroid of triangle formed by the points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

For Example: Find centroid of triangle whose vertices are (1,4), (−1,−1), (3,−2)
Solution: We know that the coordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

So, the coordinates of the centroid of a triangle whose vertices are (1, 4), (−1, −1) and (3, −2) are $\left(\dfrac{1-1+3}{3}, \dfrac{4-1-2}{3}\right) = \left(1, \dfrac{1}{3}\right)$

(9) The are of triangle formed by the points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) is $\dfrac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$ or, $\dfrac{1}{2}|(x_1y_2+x_2y_3+x_3y_1)-(x_1y_3+x_2y_1+x_3y_2)|$

For Example: Find area of triangle whose vertices are (6, 3), (−3, 5), (4, 2)
Solution: Let A = (x_1, y_1) = (6, 3), B = (x_2, y_2) = (−3, 5) and C= (x_3, y_3) = (4, −2) be the given points

Area of ΔABC= $\dfrac{1}{2}|\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\}|$

$= \dfrac{1}{2}|\{6(5-(-2)) - 3(-2-3) + 4(3-5)\}|$

$= \frac{1}{2} |\{6 \times 7 + 15 - 8\}|$

$= \frac{1}{2} |57 - 8|$

$= \frac{49}{2}$ sq. units

(10) If points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are collinear, then $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$

For Example: Let A(2,5), B(4,6) and C(8,8) be the given points.
Three points are collinear if area enclosed by three points is zero.

Area of $\Delta ABC = \frac{1}{2} |\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}|$

$= \frac{1}{2} |2\,(6 - 8) + 4(8 - 5) + 8(5 - 6)|$

$= \frac{1}{2} |2 \times (-2) + 4 \times (3) + 8 \times (-1)|$

$= \frac{1}{2} |-4 + 12 - 8|$

$= \frac{1}{2} |-12 + 12|$

$= 0$