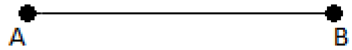
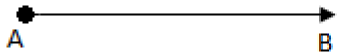
Lines and Angles

(1) Line Segment: It is a line with two end points.



It is denoted by $\overline{AB}$.

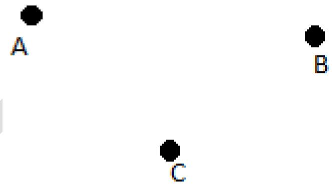
(2) Ray: It is a line with one end point.



It is denoted by $\overrightarrow{AB}$.

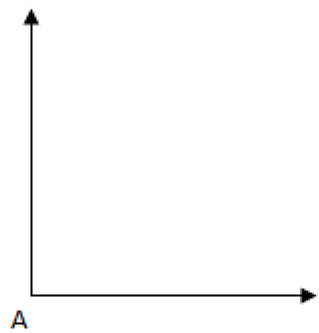
(3) Collinear Points: If three or more points lie on the same line, they are called collinear points; otherwise they are called non – collinear points.

For the figure shown above, A, B and C are collinear points.



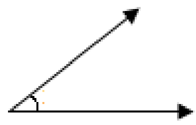
For the figure shown above, A, B and C are non – collinear points.

(4) Angle : It is formed when two rays originate from the same end point. The rays which form an angle are called its arms and the end point is called the vertex of the angle.

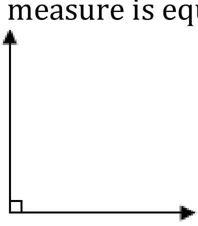


(5) Types of Angles:

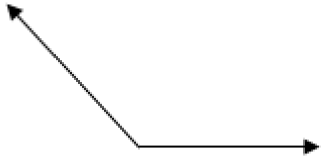
(i) Acute angle: It is the angle whose measure is between 0° and 90°.



(ii) Right angle: It is the angle whose measure is equal to 90°.

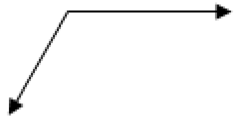


(iii) Obtuse angle: It is the angle whose measure is greater 90° than but less than 180°.

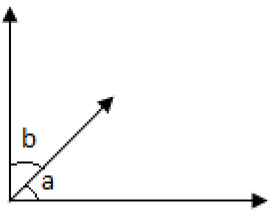


(iv) Straight angle: It is the angle whose measure is equal to 180°.

(v) Reflex angle: It is the angle whose measure is greater 180° than but less than 360°.

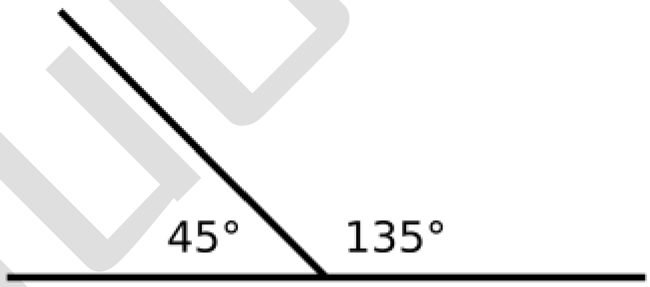


(vi) Complementary angles: The two angles whose sum is 90° are known as complementary angles.



For the figure shown above, the sum of angles a & b is 90°, hence these two angles are complementary angles.

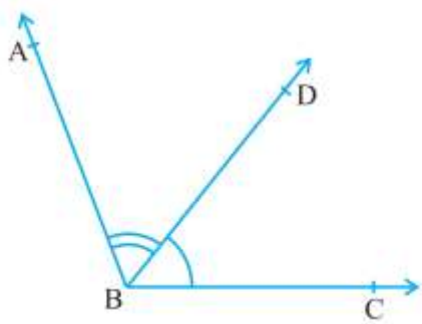
(vii) Supplementary angles: The two angles whose sum is 180° are known as supplementary



angles.

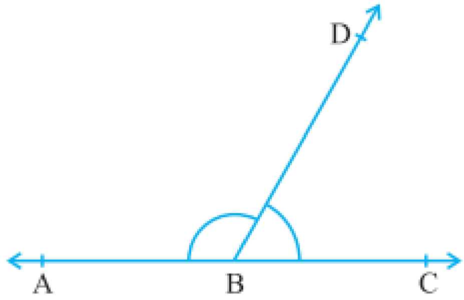
For the figure shown above, sum of the angles 45° and 135° is 180°, hence these two angles are supplementary angles.

(viii) Adjacent angles: Two angles are said to be adjacent if they have a common vertex, a common arm and their non-common arms are on different side of the common arm.
When two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms.



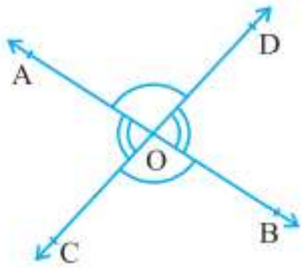
For the figure shown above, ∠ ABD and ∠ DBC are adjacent angles. Here, ray BD is the common arm and B is the common vertex. And ray BA and BC are non-common arms.
Here, ∠ ABC = ∠ ABD + ∠ DBC.

(ix) Linear pair of angles: Two angles are said to be linear if they are adjacent angles formed by two intersecting lines. The linear pair of angles must add up to 180°.



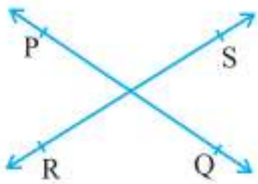
For the figure shown above, ∠ ABD and ∠ DBC are called linear pair of angles.

(x) Vertically Opposite angles: These are the angles opposite each other when two lines cross.



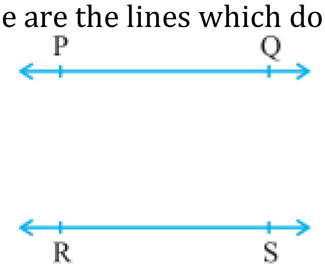
For the figure shown above, ∠ AOD and ∠ BOC are vertically opposite angles. Also, ∠ AOC and ∠ BOD are vertically opposite angles.

(xi) Intersecting lines: These are the lines which cross each other.



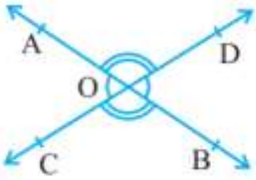
For the figure shown above, lines PQ and RS are the intersecting lines.

(xii) Non-intersecting lines: These are the lines which do not cross each other.



For the figure shown above, lines PQ and RS are the non-intersecting lines.

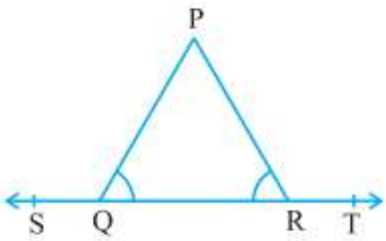
(6) Pair of Angles:
Axiom 1: If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.
Axiom 2: If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.
Theorem 1: If two lines intersect each other, then the vertically opposite angles are equal.
Proof:



Suppose AB and CD are two lines intersecting each other at point O.
Here, the pair of vertically opposite angles formed are (i) ∠ AOC and ∠ BOD (ii) ∠ AOD and ∠BOC And we need to prove that ∠ AOC = ∠ BOD and ∠ AOD = ∠ BOC.
Here, ray OA stands on line CD. Hence, ∠ AOC + ∠ AOD = 180° as per linear pair axiom. Similarly, ∠ AOD + ∠ BOD = 180°.
On equating both, we get, ∠ AOC + ∠ AOD = ∠ AOD + ∠BOD
Thus, ∠ AOC = ∠BOD
similarly, it can be proved that ∠AOD = ∠BOC.

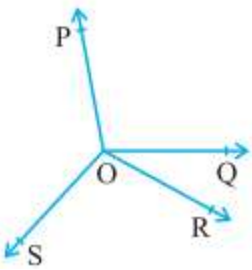
(7) Some Examples:

For Example: ∠ PQR = ∠ PRQ, then prove that ∠ PQS = ∠ PRT.

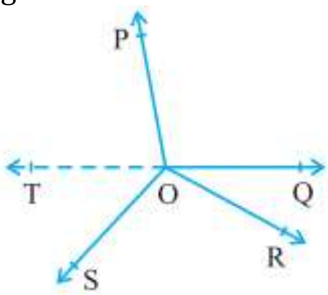


From the figure, we can see that ∠ PQS and ∠ PQR forms a linear pair.
Hence, ∠ PQS +∠ PQR = 180° i.e. ∠ PQS = 180° – ∠ PQR – (i)
Also, from the figure, we can see that ∠ PRQ and ∠ PRT forms a linear pair.
Hence, ∠ PRQ +∠ PRT = 180° i.e. ∠ PRT = 180° – ∠ PRQ
Given, ∠ PQR = ∠ PRQ
Therefore, ∠ PRT = 180° – ∠ PQR – (ii)
From (i) and (ii),
∠ PQS = ∠ PRT = 180° – ∠ PQR
Thus, ∠ PQS = ∠ PRT

For Example: OP, OQ, OR and OS are four rays. Prove that ∠ POQ + ∠ QOR + ∠ SOR + ∠ POS = 360°.



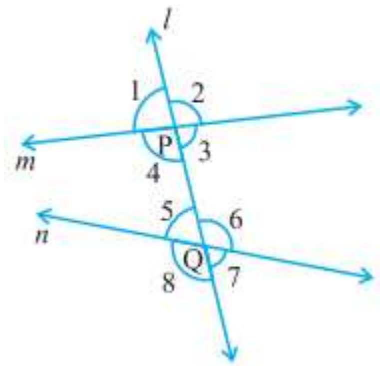
Firstly, let us make ray OT as shown in figure below to make a line TOQ.



From the above figure, we can see that, ray OP stands on line TOQ.
Hence, as per linear pair axiom, $\angle TOP + \angle POQ = 180°$ – (i)
Similarly, from the figure, we can see that, ray OS stands on line TOQ.
Hence, as per linear pair axiom, $\angle TOS + \angle SOQ = 180°$
But, from the figure, $\angle SOQ = \angle SOR + \angle QOR$
So, $\angle TOS + \angle SOR + \angle QOR = 180°$ – (ii)
On adding (i) & (ii), we get,
$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360°$
From the figure, $\angle TOP + \angle TOS = \angle POS$
Therefore, $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$.

(8) Parallel lines and a Transversal:

Transversal: It is a line which intersects two or more lines at distinct points.



Here, line *l* intersects lines *m* and *n* at P and Q respectively. Thus, line *l* is transversal for lines *m* and *n*.

(a) Exterior angles: These are the angles outside the parallel lines.
Here, $\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$ are exterior angles.

(b) Interior angles: These are the angles inside the parallel lines.
Here, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ are interior angles.

(c) Corresponding angles: These are angles in the matching corners.
Here, (i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$ (iii) $\angle 4$ and $\angle 8$ (iv) $\angle 3$ and $\angle 7$ are corresponding angles.

(d) Alternate interior angles: The angles that are formed on opposite sides of the transversal and inside the two lines are alternate interior angles.
Here, (i) $\angle 4$ and $\angle 6$ (ii) $\angle 3$ and $\angle 5$ are alternate interior angles.

(e) Alternate exterior angles: The angles that are formed on opposite sides of the transversal and outside the two lines are alternate exterior angles.
Here, (i) $\angle 1$ and $\angle 7$ (ii) $\angle 2$ and $\angle 8$ are alternate exterior angles.

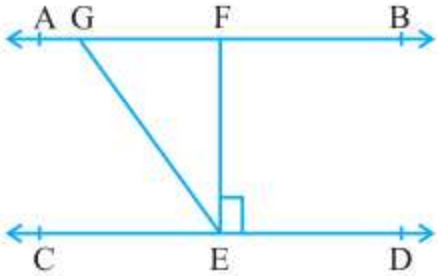
(f) Interior angles on the same side of the transversal: (i) $\angle 4$ and $\angle 5$ (ii) $\angle 3$ and $\angle 6$.They are also known as consecutive interior angles or allied angles or co-interior angles.

Axiom 1: If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
Axiom 2: If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.
Theorem 1: If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.
Theorem 2: If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.
Theorem 3: If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.
Theorem 4: If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

(9) Lines Parallel to the Same Line:

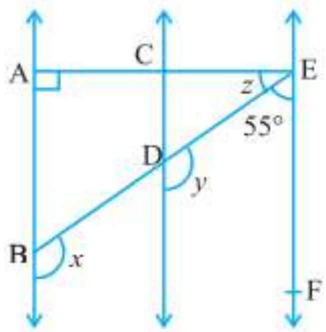
Theorem 1: Lines which are parallel to the same line are parallel to each other.

For Example: If AB || CD, EF ⊥ CD and ∠ GED = 126°, find ∠ AGE, ∠ GEF and ∠ FGE.



From the figure, we can see that, ∠ AGE and ∠ GED forms alternate interior angles.
Therefore, ∠ AGE = ∠ GED = 126°
From the figure, we can see that, ∠ GEF = ∠ GED – ∠ FED = 126° – 90° = 36°
Again from the figure, we can see that, ∠ FGE and ∠ AGE forms linear pair.
Therefore, ∠ FGE + ∠ AGE = 180°
∠ FGE = 180° – 126° = 54°.

For Example: AB || CD and CD || EF. Also, EA ⊥ AB. If ∠ BEF = 55°, find the values of x, y and z.



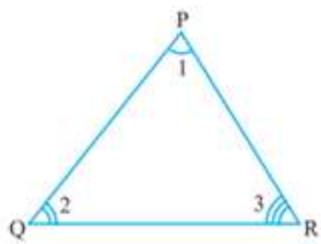
From the figure, we can see that, ∠ y and ∠ DEF forms interior angles on the same side of the transversal ED.
Therefore, y + 55° = 180° => y = 180° – 55° = 125°
From the figure, we can see that, AB || CD, so as per corresponding angles axiom x = y.
So, x = 125°
From the figure, we can see that, AB || CD and CD || EF, hence, AB || EF.
Therefore, ∠ EAB + ∠ FEA = 180° – (i)
From the figure, ∠ FEA = ∠ FEB + ∠ BEA.
Substituting in (i), we get,
∠ EAB + ∠ FEB + ∠ BEA = 180°
90° + z + 55° = 180° i.e. z = 35°.

(10) Angle Sum Property of a Triangle:

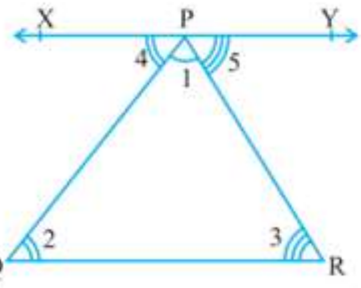
Theorem 1: The sum of the angles of a triangle is 180°.

Theorem 2: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Proof:

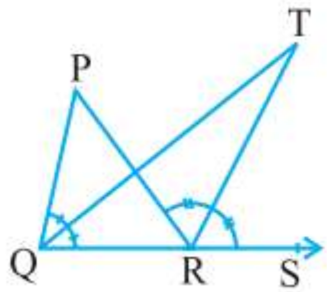


For the given triangle PQR, we need to prove that $\angle 1 + \angle 2 + \angle 3 = 180°$.
Firstly, we will draw line XPY parallel to QR passing through P as shown in figure below.



From the figure, we can see that $\angle 4 + \angle 1 + \angle 5 = 180°$ – (1)
Here, XPY || QR and PQ, PR are transversals. So, $\angle 4 = \angle 2$ and $\angle 5 = \angle 3$ (Pairs of alternate angles).
Substituting $\angle 4$ and $\angle 5$ in (1), we get, $\angle 1 + \angle 2 + \angle 3 = 180°$.
Hence, the sum of the angles of a triangle is 180°.

For Example: The side QR of Δ PQR is produced to a point S. If the bisectors of $\angle$ PQR and $\angle$ PRS meet at point T, then prove that $\angle QTR = 1/2 \angle QPR$.



We know that, the exterior angle of triangle is equal to the sum of the two interior angles.
So, $\angle TRS = \angle TQR + \angle QTR$ i.e. $\angle QTR = \angle TRS - \angle TQR$ – (i)
Similarly, $\angle SRP = \angle QPR + \angle PQR$ – (ii)
From the figure, $\angle SRP = 2 \angle TRS$ and $\angle PQR = 2 \angle TQR$
Hence, equation (ii) becomes,
$2 \angle TRS = \angle QPR + 2 \angle TQR$
$\angle QPR = 2 \angle TRS - 2 \angle TQR \Rightarrow \frac{1}{2} \angle QPR = \angle TRS - \angle TQR$ – (iii)
On equating (i) and (iii), we get,
$\angle QTR = \frac{1}{2} \angle QPR$.