Euclid's Geometry

(1) Geometry:
The word 'geometry' originates from Greek words 'geo' which means Earth and 'metrein' which means measure.

(2) Surface:
The boundaries of solids having shape, size, position and which is movable are called surfaces. They usually have no thickness and separate one part of space from another. The boundaries of these surface can be curved or a straight line. The lines end in points.

(3) Dimension:
It is a measurable extent of anything like length, breadth, and depth/height. A solid has three dimensions, a surface has two, a line has one and a point has no dimensions.

(4) Euclid's Definitions:
(i) A point is one which has no part.
(ii) A line is breadth less length.
(iii) The ends of a line are points.
(iv) A straight line is a line which lies evenly with the points on itself.
(v) A surface is that which has length and breadth only.
(vi) The edges of a surface are lines.
(vii) A plane surface is a surface which lies evenly with the straight lines on itself.

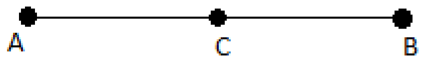
(5) Euclid's Axioms:
(i) Things which are equal to the same thing are equal to one another.
(ii) If equals are added to equals, the wholes are equal.
(iii) If equals are subtracted from equals, the remainders are equal.
(iv) Things which coincide with one another are equal to one another.
(v) The whole is greater than the part.
(vi) Things which are double of the same things are equal to one another.
(vii) Things which are halves of the same things are equal to one another.
Axiom 1: Given two distinct points, there is a unique line that passes through them.

(6) Euclid's Postulates:
Postulate 1: A straight line may be drawn from any one point to any other point.
Postulate 2: A terminated line can be produced indefinitely.
Postulate 3: A circle can be drawn with any centre and any radius.
Postulate 4: All right angles are equal to one another.
Postulate 5: If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

(7) Consistent:
A system of axioms is called consistent.

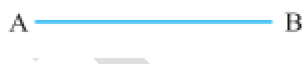
(8) Euclid's Theorem:
Two distinct lines cannot have more than one point in common.
Proof:
(i) Suppose that we have two lines x and y. And we need to prove that they have only one common point.
(ii) We shall assume that the lines x and y intersect at two distinct points A & B.
(iii) Hence, we have two lines passing through two distinct points A & B. But, this contradicts the Euclid's axiom that only one line can pass through two distinct points.
(iv) Thus, our assumption that two lines can pass through two distinct points is wrong.
(v) Hence, two distinct lines cannot have more than one point in common.

For Example: If a point C lies between two points A and B such that AC = BC, then prove that AC = 1/2 AB. Explain by drawing the figure.



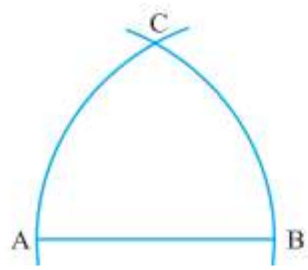
Given, AC = BC.
Adding AC on both the sides, we get,
AC + AC = BC + AC – (i)
From the figure, we can see that, AB is the line segment part of two parts AC and BC.
Thus, AB = AC + BC – (ii)
Substituting in (i), we get,
AC + AC = AB i.e. 2AC = AB
Hence, AC = ½ AB.

For Example: Prove that an equilateral triangle can be constructed on any given line segment.
(i) Consider a line segment AB as shown below:

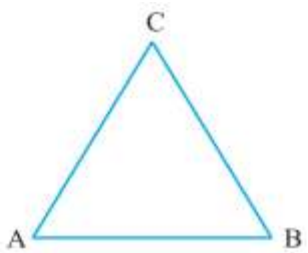


(ii) Using Euclid's postulate 3, we draw two circles one keeping A as centre and radius equal to AB. And another circle with B as centre and radius equal to BA.

(iii) Let C be the point where these circles meet.



(iv) Now, draw line segments AC to BC to complete the triangle ABC.

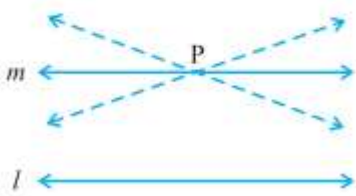


(v) Here, while drawing, we chose radii AC = AB and radii BC = AB.

(vi) Hence, AB = BC = CA which means Δ ABC is an equilateral triangle.

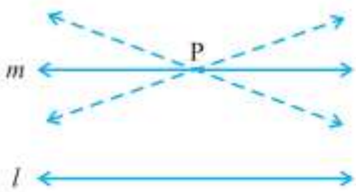
(9) Equivalent Versions of Euclid's Fifth Postulate:

'For every line *l* and for every point P not lying on *l*, there exists a unique line *m* passing through P and parallel to *l*'.



Or in other words, 'Two distinct intersecting lines cannot be parallel to the same line'.

For Example: Consider the following statement: There exists a pair of straight lines that are everywhere equidistant from one another. Is this statement a direct consequence of Euclid's fifth postulate? Explain.



(i) Consider any line *l*. And take point P anywhere but not on line *l*.

(ii) Now, as per Euclid's fifth postulate, there will be a unique line *m* passing through P which is parallel to *l*.

(iii) The distance of any point on line *m* is the length of perpendicular distance from point to the line *l*. The distance for each point on line *m* will be same from line *l*. Hence, there exists pair of straight lines equidistant everywhere from one another.