Quadratic Equations

(1) A polynomial of degree 2 is called a quadratic polynomial. The general form of a quadratic polynomial is $ax^2 + bx + c$, where a, b, c are real number such that $a \neq 0$ and x is a real variable.

For Example: $x^2 + 5x + 3$, where a = 1, b = 5, c = 3 are real number. So given equation is quadratic polynomial.

(2) If $p(x) = ax^2 + bx + c$, $a \neq 0$ is a quadratic polynomial and α is a real number, then $p(\alpha) = a\alpha^2 + b\alpha + c$ is known as the value of the quadratic polynomial p(α)

For Example: $p(\alpha) = \alpha^2 + 5\alpha + 3$ in that equation if α = 3 then p(α) = 27. So 27 is a value of quadratic polynomial

(3) A real number α is said to be a zero of quadratic polynomial $p(x) = ax^2 + bx + c$, if $p(\alpha) = 0$.

For Example: $p(x) = x2 + 6x + 5$

If $x = (-5)$ then $p(x) = 0$, So −5 is the zero of polynomial.

(4) If $p(x) = ax^2 + bx + c$ is a quadratic polynomial, then $p(x) = 0$ i.e., $ax^2 + bx + c = 0$, $a \neq 0$ is called a quadratic equation.

For Example: $p(x)=x^2-8x+16$ is a quadratic polynomial, then $p(x)=0$ i.e., $x^2 -8x + 16 = 0$, $a \neq 0$ is called a quadratic equation.

(5) A real number α is said to be a root of the quadratic equation $ax2+bx+c=0$. In other words, α is a root of $ax^2 + bx + c = 0$ if and only if α is a zero of the polynomial $p(x) = ax^2 + bx + c$.

For Example: Suppose quadratic equation is $2x^2 - x - 6 = 0$.

If we put $x = 2$ then $p(x) = 0$, So 2 is a root of that given equation so here α = 2.

(6) If $ax^2 + bx + c = 0$, $a \neq 0$ is factorizable into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.

For Example: The Given equation is $9x^2 - 3x - 2 = 0$

Now, Solving the above equation using factorization method.

=> $9x^2 - 6x + 3x - 2$

=> $3x(3x - 2) + 1(3x - 2)$

=> $(3x + 1)(3x - 2) = 0$

=> $(3x + 1) = 0$ or $(3x - 2) = 0$

=> $3x = -1$ or $3x = 2$

=> $x = \frac{-1}{3}$ or $x = \frac{2}{3}$

Hence, $x = \frac{-1}{3}$ and $x = \frac{2}{3}$ are the two roots of the given equation

(7) The roots of a quadratic equation can also be found by using the method of completing the square.

For Example: The Given equation is –

$2x^2 - 7x + 3 = 0$
Dividing throughout by 2

$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

Shifting the constant term to the right hand side.

$\Rightarrow x^2 - \frac{7}{2}x = -\frac{3}{2}$

Adding square of the half of coefficient of x on the both side.

$\Rightarrow x^2 - \frac{7}{2}x + (\frac{7}{4})^2 = -\frac{3}{2} + (\frac{7}{4})^2$

$\Rightarrow x^2 - 2\frac{7}{4}x + (\frac{7}{4})^2 = -\frac{3}{2} + (\frac{49}{16})$

$\Rightarrow (x - \frac{7}{4})^2 = \frac{-24+49}{16}$

$\Rightarrow (x - \frac{7}{4})^2 = \frac{25}{16}$

Taking square root of both sides

$\Rightarrow (x - \frac{7}{4})^2 = \sqrt{\frac{25}{16}}$

$\Rightarrow (x - \frac{7}{4}) = \pm\frac{5}{4}$

$\Rightarrow x = \frac{5}{4} + \frac{7}{4} \text{ or } x = -\frac{5}{4} + \frac{7}{4}$

$\Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4}$

$\Rightarrow x = 3 \text{ or } x = \frac{1}{2}$

Hence x= 3, and x=1/2 are the two root of the given equation

(8) The roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ can be found by using the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided that $\sqrt{b^2 - 4ac} \geq 0$.

For Example: $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$ the given equation in the form of $ax^2 + bx - c = 0$,
Where a= √3, b = 10, c = 8√3
Therefore, the discriminant
D = b^2 - 4ac
D = $(10)^2$ – 4 x √3 x (-8√3)
D= 100 + 96
D= 196
Since, D > 0
Therefore, the roots of the given equation are real and distinct.
The real roots α and β are given by,

$$\alpha = \frac{-b+\sqrt{D}}{2a}$$

$$\alpha = \frac{-10+\sqrt{196}}{2\sqrt{3}}; \alpha = \frac{-10+14}{2\sqrt{3}}$$

$$\alpha = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

For,

$$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-10-\sqrt{196}}{2\sqrt{3}}$$

$$\beta = \frac{-10-14}{2\sqrt{3}} = \frac{-24}{2\sqrt{3}}$$

$$\beta = \frac{-12}{\sqrt{3}} = \frac{-4x\sqrt{3}x\sqrt{3}}{\sqrt{3}} = -4\sqrt{3}$$

Hence $\alpha = \frac{2}{\sqrt{3}}$ and $\beta = -4\sqrt{3}$ are the two root of the given equation.

(9) Nature of the roots of quadratic equation ax^2 + bx + c = 0, a ≠ 0 depends upon the value of D = b^2 – 4ac, which is known as the discriminate of the quadratic equation.

For Example: Value of D can be **(i)** D > 0, **(ii)** D = 0 **(iii)** D < 0.

(10) The quadratic equation ax^2 + bx + c = 0 , a ≠ 0 has:
(i) Two distinct real roots, if D ba-4ac 0 two equal roots i.e. coincident real roots if D = b^2 – 4ac > 0

For Example: $16x^2 = 24x + 1$
$16x^2 - 24x - 1 = 0$
The given equation is of the form of ax^2 + bx + c = 0, where a = 16, b = -24, c = -1
Therefore, the discriminant D = b^2 – 4ac
D= $(-24)^2 - 4 \times 16 \times (-1)$
D= 576 + 64
D= 640
Since, D > 0
Therefore, the roots of the given equation are real and distinct.
The real roots α and β are given by,

$$\alpha = \frac{-b+\sqrt{D}}{2a}$$

$$\alpha = \frac{-(-24)+\sqrt{640}}{2x16}$$

$$\alpha = \frac{24+\sqrt{64x10}}{32}$$

$$\alpha = \frac{24+8\sqrt{10}}{32}$$

$\alpha = 8(\frac{3+\sqrt{10}}{32})$

$\alpha = (\frac{3+\sqrt{10}}{4})$

For, $\beta = \frac{-b-\sqrt{D}}{2a}$

$\beta = \frac{-(-24)-\sqrt{640}}{2x16}$

$\beta = \frac{24-\sqrt{64x10}}{32}$

$\beta = \frac{24-8\sqrt{10}}{32}$

$\beta = 8(\frac{3-\sqrt{10}}{32})$

$\beta = (\frac{3-\sqrt{10}}{4})$

Hence $\alpha = (\frac{3+\sqrt{10}}{4})$ and $\beta = (\frac{3-\sqrt{10}}{4})$ are the two root of the given equation.

(ii) Two equal roots i.e. coincident real roots, if D = $b^2 - 4ac = 0$.

For Example: $2x^2 - 2\sqrt{6}x + 3 = 0$

The given equation is of the form of $ax^2 + bx + c = 0$, where a = 2, b = – 2√6, c = 3

Therefore, the discriminant D = $b^2 - 4ac$

= $(-2\sqrt{6})^2 - 4 \times 2 \times 3$

= 24 – 24

= 0

Since, D = 0

Therefore, the roots of the given equation are real.

The real and equal roots are given by $\frac{-b}{2a}$ and $\frac{-b}{2a}$

$\frac{-b}{2a} = \frac{-(-2\sqrt{6})}{2x2} = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$

$\Rightarrow \Rightarrow \frac{\sqrt{3}\sqrt{3}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$

(iii) No real roots, if $D = b2 - 4ac < 0$.

For Example: The given equation is

$x^2 + x + 2 = 0$

The given equation is of the form of $ax^2 + bx + c = 0$, where a = 1, b = 1, c = 2

Therefore, the discriminant

$D = b^2 - 4ac$

$D = (1)^2 - 4 \times 1 \times 2$

$D = 1 - 8$

$D = -7$

Since, $D < 0$

Therefore, the given equation has not real roots.