Number System

(1) Types of Numbers:

(i) Natural numbers: The group of the positive numbers which are countable are known as natural numbers. We denote natural numbers by symbol **N**.
For example: 1, 2, 3, 4, 5, etc. are some natural numbers.

(ii) Whole numbers: The group of natural numbers with inclusion of zero in it are known as whole numbers. We denote whole numbers by symbol **W**.
For example: 0, 1, 2, 3, 4, 5, etc. are some whole numbers.

(iii) Integer numbers: The group of positive and negative numbers along with zero are known as integer numbers. We denote whole numbers by symbol **Z**. The symbol Z for integers comes from the word "zahlen" which means to count.
For example: -3, -2, -1, 0, 1, 2, 3, 4, 5, etc. are some integer numbers.

(iv) Rational numbers: The numbers which can be expressed as ratio of integers are known as rational numbers. In other words, these are the numbers which can be expressed in p/q form; where $q \neq 0$.We denote whole numbers by symbol **Q**. The word 'Rational' comes from the word 'ratio' and symbol Q comes from 'quotient'.
For example: 1/4, 2/7, – 3/10, 34/7, etc. are some rational numbers.
Note: All the other types of number can be expressed as rational numbers.

(2) Equivalent rational numbers/factors:

A rational number does not have unique representation.
Let us take an example to understand them:
We can write, 1/3 = 2/6 = 4/12 = 12/36 and the list goes on. So, these types of numbers are known as equivalent rational numbers. However, for any p/q rational number, we assume that p and q have no common factors other than 1.
Remember: There are infinite rational numbers between any two given rational numbers.
For example: Find five rational numbers between 2/7 and 8/7.
(i) We know that, there are infinite rational numbers between any two given rational numbers.
(ii) Here, we can write 2/7 = (2×2) / (7×2) = 4/14 and 8/7 = (8×2) / (7×2) = 16/14.
(iii) Now, five rational numbers between 2/7 and 4/7 are 5/14, 6/14, 7/14, 8/14, 9/14.

(3) Irrational Numbers:

Irrational numbers are the numbers which cannot be written in p/q form, where p and q are integers and $q \neq 0$. The irrational numbers were discovered by the Pythagoreans. There are indefinite irrational numbers.
For example: √2, √3, √15, π, 0.101010110010001…..etc. are some irrational numbers.

(4) Real numbers:

The group of numbers which includes rational and irrational numbers in it are known as real numbers. We denote real numbers by symbol **R**.

Fact – Two German Mathematicians showed that: Corresponding to every real number, there is a point on the real number line, and corresponding to every point on the number line, there exists a unique real number.

(5) Pythagoras theorem to locate an irrational number √n on the real number line:

(i) Steps to locate irrational number:

(i) Step 1: Find the Pythagorean triplet for given √n. Let x and y be the two other Pythagorean triplets than √n (Assume $x > y$).

(ii) Step 2: Out of x and y, locate from origin (O) the point which is larger x in this case on the real number line.

(iii) Step 3: Draw from x a perpendicular line segment of length y units.

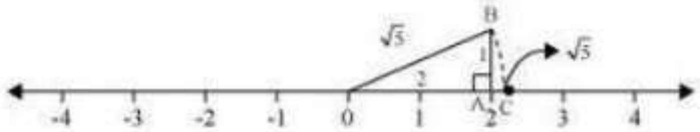
(iv) Step 4: Draw an arc of radius Oy on the number line. The point where this arc will intersect represents √n.

For Example: Locate √5 on the number line.

(i) Firstly, we will find the other two numbers whose result will be √5 satisfying the Pythagoras theorem.

(ii) In this case $\sqrt{(2)^2} + \sqrt{(1)^2} = \sqrt{5}$.

(iii) Now, draw a number line. Mark point A which will be 2 units from origin. Then draw perpendicular line segment AB of unit length. Take origin as centre and OB as radius; draw an arc intersecting number line at C.

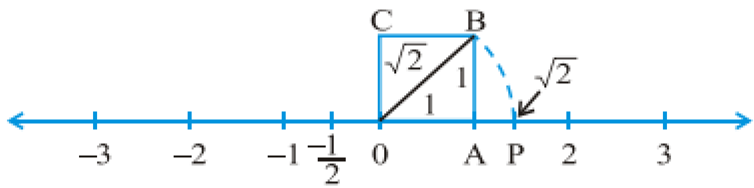


(iv) In the figure, OC represents √5.

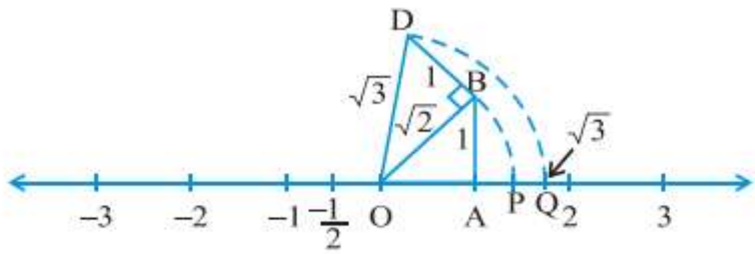
(ii) Locating √n point on number line for already drawn √n-1:

For Example: Locate √3 on the number line.

(i) In this case, we will locate √2 on number as shown in the above example.



(ii) For already drawn √2, draw unit perpendicular length BD to OB. Now, keeping O as centre draw an arc from point D which will intersect the number line at Q.



(iii) In the figure, OQ represents √3.

(6) Real Numbers and their decimal expansions:

When a rational number is divided, following are some points which could be noted:

(i) A remainder becomes 0 or remainder repeats itself after certain stage.
(ii) The number of entries in the repeating string of remainders is less than the divisor.
(iii) If the remainders repeat, then we get a repeating block of digits in the quotient.

Following are the two major cases:

Case 1: The remainder becomes zero.

This includes those rational numbers whose remainder terminates or ends after a finite number of steps. The decimal expansion of such numbers is called as terminating.

For example: ½ = 0.5, 7/8 = 0.875, etc. are some terminating real numbers.

Case 2: The remainder never becomes zero.

This includes those rational numbers whose remainder does not terminate or ends. The decimal expansion of such numbers is called non-terminating or recurring.

For example: 1/3 = 0.3333…., 1/7 = 0.142857142857….., etc. are some non – terminating real numbers.

Some Examples:

***For Example*:** Write decimal form and state the type of decimal expansions for following numbers:

(a) 1/11

(i) 1/11 = 0.0909090… = $0.\overline{09}$

(ii) Type: Non-terminating

(b) 329/400

(i) 329/400 = 0.8225

(ii) Type: Terminating

***For Example*:** Express 0.6666…. in the p/q form.

(i) Let x = 0.6666…

(ii) So, 10x = 6.666…

(iii) Thus, 10x = 6 + x i.e. 9x = 6

(iv) x = 2/3.

(v) Hence, p/q form of 0.6666…. is 2/3.

(7) Finding irrational numbers between two numbers:

***For Example*:** Find three irrational numbers between 5/7 and 9/11.

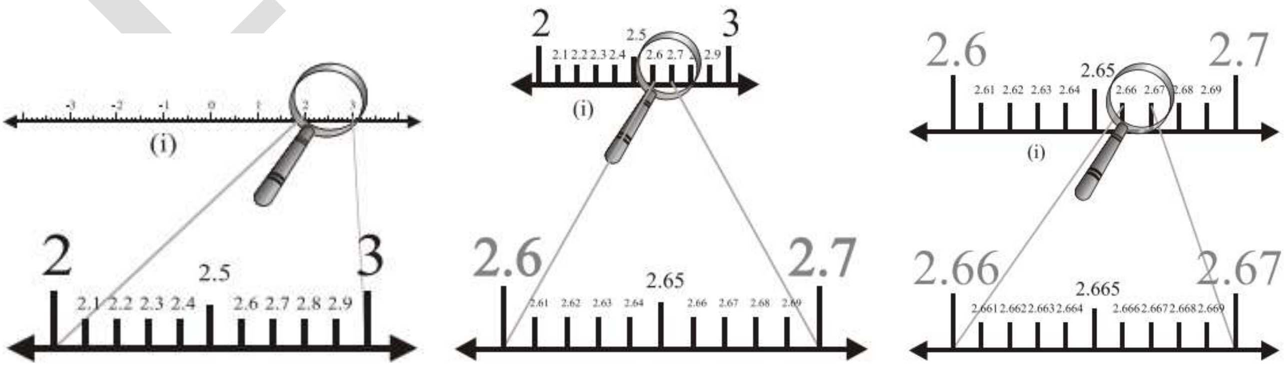
(i) Here, 5/7 = $0.\overline{714285}$ and 9/11 = $0.\overline{81}$

(ii) In between these range, three irrational numbers can be 0.7201272012…, 0.7301373013… and 0.7401474014…..

(8) Representing Real numbers on the number line:

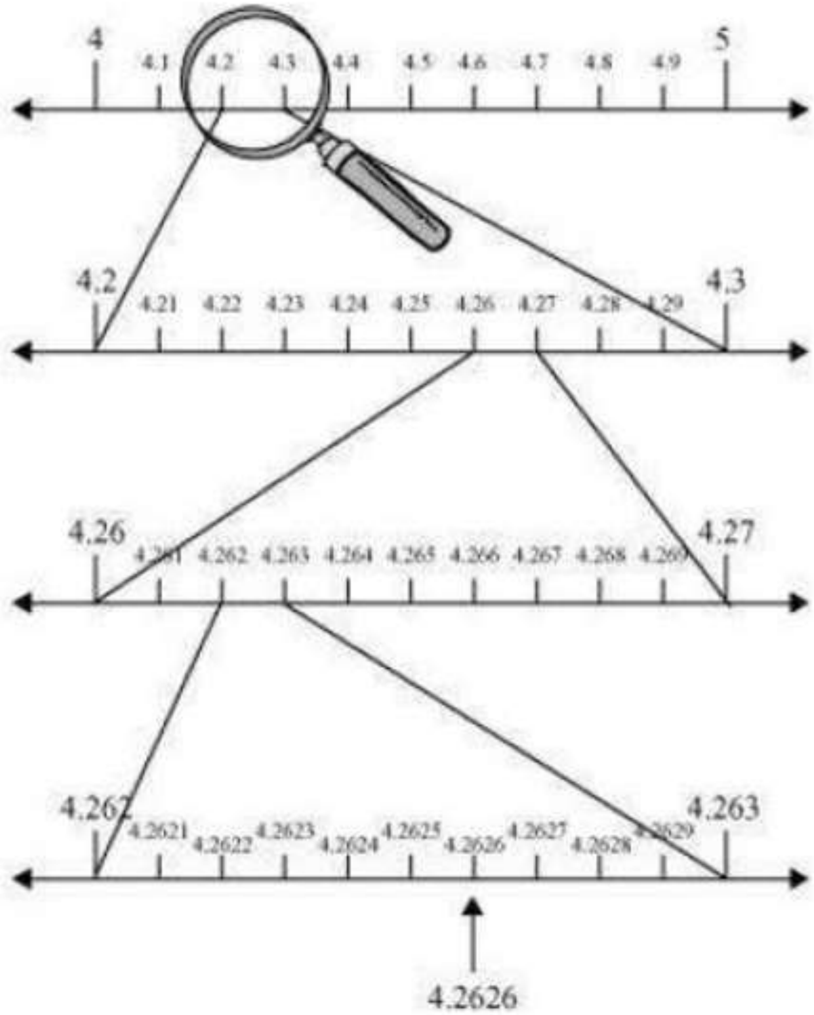
Look at figures below to understand number line in detail:

Here, we are needed to locate 2.665 on the number line.



The figures here represent the process of visualisation of representation of numbers on the number line. Such process is known as the process of successive magnification.

For Example: Visualise $4.\overline{26}$ on the number line up to 4 decimal places.
(i) We know that, $4.\overline{26}$ = 4.262626……
(ii) Here, we need to represent $4.\overline{26}$ up to 4 decimal places. So, we need to represent 4.2626 on the number line.



(9) Operations on real numbers:

For any two real numbers x and y, following identities hold true:

(i) $\sqrt{xy} = \sqrt{x}\sqrt{y}$

(ii) $\sqrt{x/y} = \dfrac{\sqrt{x}}{\sqrt{y}}$

(iii) $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y$

(iv) $(x + \sqrt{y})(x - \sqrt{y}) = x^2 - y$

(v) $(\sqrt{x} + \sqrt{y})(\sqrt{a} + \sqrt{b}) = \sqrt{xa} + \sqrt{xb} + \sqrt{ya} + \sqrt{yb}$

(vi) $(\sqrt{x} + \sqrt{y})^2 = x + 2\sqrt{xy} + y$

For Example: Simplify $(3+\sqrt{3})\,(2+\sqrt{2})$.

$=(3+\sqrt{3})\,(2+\sqrt{2}) = 3 \times (2+\sqrt{2}) + \sqrt{3} \times (2+\sqrt{2})$

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

For Example: Rationalise the denominator of $1 / (\sqrt{7} - 2)$.
$1 / (\sqrt{7} - 2) = 1 / (\sqrt{7} - 2) \times (\sqrt{7} + 2) / (\sqrt{7} + 2)$
$= (\sqrt{7} + 2) / ((\sqrt{7})^2 - (2)^2)$
$= (\sqrt{7} + 2) / (7 - 4)$
$= (\sqrt{7} + 2)/3$

(10) Laws of Exponents for Real Numbers:

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $a^m / a^n = a^{m-n}$
4. $a^m \times b^m = (ab)^m$

For Example: Find $64^{1/2}$.
(i) $64^{1/2} = (2^6)^{1/2}$
(ii) Using property $(a^m)^n = a^{mn}$, we can write,
(iii) $64^{1/2} = 2^{6 \times 1/2}$
$= 2^3 = 8$

For Example: Simplify $7^{1/2} \times 8^{1/2}$.
(i) $7^{1/2} \times 8^{1/2} = (7 \times 8)^{1/2}$
(ii) Using property $a^m \times b^m = (ab)^m$, we can write,
(iii) $7^{1/2} \times 8^{1/2} = (56)^{1/2}$